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A SEMI-EMPIRICAL METHODOLOGY FOR BALANCED FIELD LENGTH ESTIMATION OF JET-ENGINED AIRCRAFT IN EARLY DESIGN PHASES

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ABSTRACT

Current levels of competitiveness displayed in business and commercial aviation market led to increasingly stringent performance and economy requirements. One of the key elements of these requirements is field performance, a factor that has great influence on the viability of certain route or operation for the aircraft in question, and that might shift the balance in a purchase decision. During early design phases, aerodynamic data about the aircraft being developed is often inaccurate and subject to changes during its evolution, which, alongside with difficulties do validate the results, renders numerical simulation methods unpractical for estimating field performance. These factors stimulated the development of a number of semi-empirical methodologies to estimate takeoff field lengths, of which some, by taking advantage of the available historical trend, produce very reasonable results and are widespread adopted on the aviation industry. Aiming to enable leaner aircraft designs, this paper presents an overview of several established methods, analyzing structure and comparing results obtained by their application to a databank of existing aircrafts. Finally, it proposes a reviewed and modified method that includes new parameters of the designed aircraft and updated calibrations, showing that it is possible to obtain relevant results, improving estimations precision and accuracy.

NOMENCLATURE

Symbols:

b	Aircraft wingspan, m (SI) or ft (Imperial)
C_D	Drag coefficient
C_L	Lift coefficient
C_{Lmax}	Maximum C_L of the aircraft, for the given aerodynamic configuration
C_{L2}	C_L of the aircraft at V_2; typically 0.694 C_{Lmax}
F_{acc}	Acceleration during takeoff, N (SI) or lbf (Imperial)
g	Standard gravity, 9.8 m/s² (SI) or 1 lbf/lb (Imperial)
h_{TO}	Takeoff screening height, 10.7 m (SI) or 35 ft (Imperial)
L/D	Lift to drag ratio; (C_L/C_D)
N_{eng}	Aircraft number of engines
S	Aircraft wing area, m² (SI) or ft² (Imperial)
S_{TOFL}	Required TOFL, m (SI) or ft (Imperial)
T	Aircraft thrust, N (SI) or lbf (Imperial)
T_4	Engine turbine inlet temperature, K
V_s	1g stall speed, knots
V_{ef}	Velocity of engine failure, knots
V_1	Decision speed, knots
V_R	Rotation speed, knots
V_{LO}	Liftoff speed, knots

V_2	Takeoff safety speed, knots
V_{mc}	Velocity of minimum control, knots
V_{mu}	Velocity of minimum unstick, knots
W	Aircraft weight, N (SI) or lbf (Imperial)
Z_{eng}	Engine centerline height to ground

Symbols with an overline at the top denote average or mean values.

Greek letters:

γ	Aircraft trajectory angle
Δ	Difference operator
μ	Ground friction coefficient
ρ	Local air density, kg/m³ (SI) or lbf/ft³ (Imperial)
ρ_{SL}	Sea Level standard air density, 1.225 kg/m³ (SI) or 0.0765 lb/ft³ (Imperial)
σ	Relative density of the air, compared with Sea Level standard; (ρ/ρ_{SL})

Abbreviations:

AEO	All engines operative
AR	Aircraft wing aspect ratio; (b^2/S)
BFL	Balanced field length
BPR	Engine bypass ratio
FAA	Federal Aviation Administration
FAR	FAA Regulations
FAR 25	FAA Regulations Part 25 – Transport category airplanes
MTOW	Maximum takeoff weight
OEI	One engine inoperative
SLS	Sea Level static, usually referring to thrust
TOFL	Takeoff field length

1 INTRODUCTION

Field performance is one of the key aspects of airplane design. In the very competitive commercial aircraft business of today, field performance is subjected to narrower design margins and very stringent market constraints. In addition, great uncertainties characterize the estimation of takeoff field length in early design phases, due to inaccurate data about the airplane under development and outdated methods for performance estimation. This way, incorrect sizing often takes place in the conceptual phase, leading to loss of competitiveness. For illustrational purposes, considering existing narrow body airliners close to MTOW, a 100 ft. reduction in the required takeoff field length (TOFL) could allow an equivalent increase of around 1% of the takeoff weight – which could be roughly translated in 5% more passengers. Reviewing and updating these established methods for calculation of field performance, mainly by including new parameters, better calibrations or new inputs weighting, could greatly contribute to the proper airplane sizing.

During these early phases of the aircraft design, the use of numerical simulation and integration to calculate performance is not practical, considering that it involves several aerodynamic characteristics of the airplane, which have an error margin greater than the required precision. Also, some mispredicted or unconsidered effect, such as interference drag or aerodynamic efficiency of high lift devices, can lead to largely inaccurate results, which would not be noticed without an adequate way to validate the numerics. For this reason, semi-empirical methods are historically used to this purpose, assuming that a relatively conventional design will follow historical trend, usually providing smaller deviations from actual results.

Concerning takeoff performance, there are many well established methods developed for early design phases that provide reasonable results and that have been used for a long time. Here, after a brief summary of the FAR 25 requirements for TOFL, an overview of selected TOFL estimation methods, taken from aircraft design textbooks, is presented, with both a theoretical analysis of the method's assumptions and mathematical structure and a practical evaluation, by applying it to a set of existent aircraft and comparing the results to the nominal performance data. Some of the input data for these methods consist of sensitive information, which makes this evaluation dependent of the accuracy of the utilized parameters. While most of them came from academic textbooks or aircraft manuals, there is margin for incorrect data that could induce unreal behaviors. However, overall results showed consistency, increasing results' confidence level. Also, the scope of this work has been restricted to jet-engined civil aircraft, due to the greater amount of data available, comparing to propeller aircraft, and to the significant differences of the behavior of these two types of engine.

Following this evaluation, the complementary methods included in the selected method's modifications, aiming to improve estimations precision and accuracy, are presented. Of these methods, it is analyzed its nature, the rationale behind its incorporation and how the method will fit in the final TOFL methodology. Finally, the proposed method is evaluated in a similar way than the current methods, and the results will be compared, showing that improved results are possible, while keeping the characteristics that made the original methodology well suited for early design phases.

2 TAKEOFF REQUIREMENTS OVERVIEW

There are a series of strict requirements to determine the TOFL of a FAR 25 certified aircraft, such as airliners and most business jets, and this section intends to present a brief summary of these requirements, in order to expose the amount of criteria that must be taken into account during design phases aiming at field performance improvement and to allow forthcoming mentions to specific terms related to the requirements. ESDU [11] is a good reference for the complete takeoff process.

Simplifying, FAR 25 states that the certified takeoff distance for a dry runway is the greater of either 115% of the distance of an uneventful takeoff run, from the start to the point where the aircraft is 35 feet above ground level, or the distance from the start, to a point where the critical engine of the aircraft fails, and then to the 35 feet height clearance point with one engine out, or the distance from the start, to a determined point where the critical engine of the aircraft fails, and then to the point where the aircraft is stopped after application of maximum effort braking. Since the 2 latter distances depend on the point where the engine is failed, it is considered that the critical engine fails at the speed that makes both distances equal, which is called Balanced Field Length (BFL). The critical engine is defined as the engine which failure causes a greater loss of performance during takeoff (usually, it is either of the engines further apart from the aircraft centerline). It is common for 2 and 3-engine aircraft that the one engine inoperative (OEI) situation defines the TOFL, while for 4-engine aircraft it is the 115% all engine operative (AEO) situation.

FAR 25 describes a number of takeoff speeds, on which the segments of the takeoff are based. The most significant for this work will be described following:

- V_{EF}: speed at which the critical engine is failed, allowing pilot recognition and action at V_1.
- V_1: decision speed, at which the pilot chooses to abort or to take off. It is always greater than V_{EF}. If the pilot recognizes an engine failure before V_1, he must abort takeoff. If he recognizes engine failure after V_1, he must proceed with takeoff. If the engine failure is recognized exactly at V_1, the resultant accelerate-stop or the continued takeoff distance would be equal to the BFL.

- V_{MC}: speed of minimum control. It is the minimum speed that guarantees that the pilot is able to control the aircraft after a sudden critical engine failure, keeping straight flight and not more than 5° of bank angle, and with reasonable control forces required.
- V_{MU}: speed of minimum unstick. Minimum speed that allows the aircraft to safety lift off the ground.
- V_R: rotation speed, at which the pilot starts the rotation of the aircraft to continue takeoff. It must be greater or equal to V_1 and 105% of V_{MC}. Also, it must allow reaching V_2 before reaching a 35 feet height and must not allow the aircraft to leave ground with a speed less than 105% of V_{MU}, even if rotated with the maximum practicable rate.
- V_2: speed to provide the minimum gradient climb after takeoff. It must not be less than 120% of V_S for takeoff configuration and 110% of V_{MC}.

It can be noted that the requirements are complex and not always straightforward, what makes it more difficult to foresee the full extent of the impact that one change in the design will have at the final aircraft, since the limiting factor for TOFL may be changed in several different ways, reducing the expected improvements. Next, the evaluated methods that intend to solve this question will be presented.

3 METHODS OVERVIEW

The methods evaluated in this paper will be presented next, followed by an evaluation of the results obtained with their application.

3.1 Roskam

Roskam [1] presents this methodology, which is based on studies and data available at Loftin [6], in the first part of his Airplane Design series. Several other well-known design textbooks, such as Raymer [7], also are based in the same reference and use the parameter proposed in this method. ESDU [10]**Erro! Fonte de referência não encontrada.** also derives a similar parameter. However, Roskam [1] was chosen to represent its usage due to his widespread adoption in aircraft design courses. It consists of a straightforward combination of three key characteristics of the aircraft that determine its takeoff performance: weight, speed and thrust. Pilot technique, aerodynamic drag and ground friction are also mentioned, but are not included in the method, probably due to the uncertainties associated. For FAR 25 regulated aircrafts, these characteristics are summarized by the Takeoff Parameter 25 (TOP_{25}), defined as follows:

$$\mathrm{TOP}_{25} = \frac{(\mathrm{W_{TO}}/\mathrm{S})}{\sigma\cdot \mathrm{C}_{\mathrm{L}_{\mathrm{max}}\mathrm{TO}}\cdot(\mathrm{T_{SLS}}/\mathrm{W_{TO}})} \qquad (1)$$

With this parameter, a linear regression is made for the set of aircrafts selected by the Loftin [6], and the resulting TOFL is calculated as:

$$\mathrm{S_{TOFL}} = 37.5 \cdot \mathrm{TOP}_{25} \qquad (2)$$

In equations (1) and (2), TOP_{25} dimension is lbs/ft², and S_{TOFL} is given in feet. The strength of this method lies in its simplicity and robustness, demonstrated by its application many years after its release with reasonable results. However, due to its heavily dependence of calibration, it may fail to capture evolutions of the historical trend, and a recalibration might not be the best solution because of the difficulties to gather a pool of reliable data of modern aircrafts, as mentioned previously. But it remains as one indicated method for the very first estimations of a new design.

3.2 Kroo

On his Design Course textbook, Kroo [2] describes analytically a balanced takeoff, based on a number of assumptions and approximations, and ultimately concludes that this is not the most adequate approach to this matter, since drag is very difficult to estimate (mainly during these early stages), as is V_{EF}, which is influenced by spoilers, brakes and rudder design. Therefore, he proposes a similar approach to the previously presented method, defining:

$$\mathrm{Index} = \frac{(W_{TO}/S)}{\sigma \cdot C_{L_{max}TO} \cdot (T_{0.7V_{LO}}/W_{TO})} \tag{3}$$

With Index, he proposes different 2nd degree fits for 2, 3 and 4-engine aircraft, as shown below:

$$S_{TOFL\ 2-eng} = 857.4 + 28.43 \cdot \mathrm{Index} + 0.0185 \cdot \mathrm{Index}^2 \tag{4a}$$
$$S_{TOFL\ 3-eng} = 667.9 + 26.91 \cdot \mathrm{Index} + 0.0123 \cdot \mathrm{Index}^2 \tag{4b}$$
$$S_{TOFL\ 4-eng} = 486.7 + 26.20 \cdot \mathrm{Index} + 0.0093 \cdot \mathrm{Index}^2 \tag{4c}$$

As in the previous method, Index dimension is lbs/ft^2, and S_{TOFL} is given in feet. Beyond the different fit, which uses 2 extra dimensions when compared to TOP, the other main difference is that the thrust value that is used is calculated for 0.7 of the lift off speed, which is assumed to be equal to 1.2 V_S. Also, the method provides some thrust decay versus Mach number curves for jet/turbofan engines with different bypass ratios, since the available data is usually considering static and sea-level thrust (T_{SLS}).

3.3 Kundu

Kundu [3] addresses the TOFL estimation with a similar approach to Kroo [2]: an analytical description of a takeoff run, but considering the AEO scenario. He also considers V_{LO} equal to V_2, which is considered to be 1.2 times V_S, that there is no drag change during the takeoff procedure and that the average acceleration of the aircraft (composed of thrust, drag and ground friction) must be evaluated at 0.7 V_2. Eventually, due to uncertainties on the estimation of drag and friction (along with a smaller contribution from these terms to the final result) and the OEI scenario, he also suggests the usage of a semi-empirical method based on data from Loftin [6], as shown below:

$$S_{TOFL\ 2-eng} = 37.5 \cdot \frac{(W_{TO}/S)}{C_{L_{max}TO} \cdot (T_{SLS}/W_{TO})} \tag{5a}$$
$$S_{TOFL\ 3-eng} = 28.5 \cdot \frac{(W_{TO}/S)}{C_{L_{max}TO} \cdot (T_{SLS}/W_{TO})} \tag{5b}$$
$$S_{TOFL\ 4-eng} = 25.1 \cdot \frac{(W_{TO}/S)}{C_{L_{max}TO} \cdot (T_{SLS}/W_{TO})} \tag{5c}$$

Units used are ft. and lbs., also. It is considered a sea level and ISA + 0° condition, this way it is not necessary to take into account air relative density. Apart from this, its results are influenced by the same aircraft characteristics than the other methods, even having the exact same formula for 2-engine aircraft than Roskam [1]. However, like the method proposed by Kroo [2], there is a differentiation for distinct number of engines.

3.4 Torenbeek, ’82

On his 1982 textbook, Torenbeek [4] does a detailed and comprehensive analysis of the takeoff dynamics, from performance and requirements point of view, considering both FAR Part 23 and 25. His methodology is also presented in Raymer [7] and Roskam [8]. The mathematical analysis is made distinguishing three cases: AEO takeoff, OEI continued takeoff and OEI accelerate-stop. Focusing on the balanced field length for FAR 25, the OEI scenarios are balanced analytically, producing an intricate equation for the desired TOFL, which is not of easy application for requiring a number of parameters that are not readily available at preliminary design phases. Therefore, some simplifications are proposed in order to make its usage possible. Namely, he describes an “inertia distance” that is included on the equation, corresponding to the distance covered by the aircraft between V_{EF} and V_1 (average speeds and reaction times are considered), estimates an average value for the climb gradient of the airborne phase of the takeoff and an average deceleration value for the aborted takeoff braking phase. Reuniting these simplifications, the result is the following equation:

$$S_{TOFL} = \frac{0.863}{1+2.3\cdot\Delta\gamma_2}\left(\frac{W_{TO}/S}{\rho\cdot g\cdot C_{L_2}} + h_{TO}\right)\cdot\left(\frac{1}{\overline{T}/W_{TO}-\mu'} + 2.7\right) + \frac{\Delta S_{TO}}{\sqrt{\sigma}} \tag{6}$$

Using ft. and lbs., ΔS_{TO} corresponds to the inertia distance, and is equal to 655 ft. $\Delta\gamma_2$ corresponds to the excess climb gradient OEI, i.e., the second gradient climb that exceeds the minimum required for the specified aircraft (it varies with engine number). C_{L2} corresponds to the lift coefficient at V_2. Considering V_2 equal to 1.2 V_S, it is equal to 0.694 C_{Lmax}. h_{TO} correspond to the screening height that marks the end of takeoff, 35 feet for FAR 25. μ' corresponds to a total deceleration component, and is estimated as shown in equation 7a. Finally, T corresponds to an average thrust component at $V_{LO}/\sqrt{2}$ which, for jet aircraft, is estimated as show in equation 7b, with BPR meaning engine bypass ratio.

$$\mu' = 0.02 + 0.01\cdot C_{L_{max}TO} \tag{7a}$$

$$\overline{T} = 0.75\cdot\frac{5+BPR}{4+BPR}\cdot T_{SLS} \tag{7b}$$

This method’s higher complexity is clearly visible, and this characteristic can bring both advantages and disadvantages, depending on the maturity level of the design. While it keeps the main components of the previously analyzed methodologies (wing loading, thrust to weight ratio and maximum lift coefficient have a high influence on the result), it includes engine related characteristics, second segment performance and reaction time and friction components. Even if some of these parameters are unreliable at the current design phase, the method suggests considerably robust estimations to proceed with the calculations (for second segment performance, it is suggested to aim at zero excess of climb gradient, in order to produce a best fitter design). Also, these extra parameters allow some customization of the method, by replacing these early estimations (such as second segment performance or thrust decay) with reliable data, as the design goes on, meaning that the estimation accuracy could be increasingly improved during the preliminary design.

3.5 Torenbeek, ‘13

Torenbeek [5] released a new aircraft design textbook in 2013, in which it is derived a different method to estimate TOFL. He separates the analysis of the ground run and the airborne segment of the takeoff. For the ground run, it is assumed that C_{L2} must be used in a parameter similar to the already familiar one presented in Loftin [6]. It is included also a parameter to take into account the variation of thrust, friction and aerodynamic drag. For the airborne distance, it is stated the uncertainties of drag and thrust variations, and that it was considered as a maneuver following a circular path followed by a steady climb. This analysis results in a similar result to what would be a steady climb to a height equal to twice the obstacle height (i.e., 70 ft.). Adding the two segments, it is obtained the following equation:

$$\mathrm{S_{TOFL}} = \frac{(\mathrm{W_{TO}/S})}{\rho \cdot \mathrm{g} \cdot \mathrm{C_{L_2}} \cdot \mathrm{k_T} \cdot (\mathrm{T_{V_2}/W_{TO}})} + 2 \cdot \mathrm{h_{TO}} \cdot \left\{ \frac{(1-\mathrm{N_{eng}^{-1}}) \cdot \mathrm{T_{V_2}}}{\mathrm{W_{TO}}} - \left(\frac{\mathrm{C_D}}{\mathrm{C_L}}\right)_{\mathrm{V_2}} \right\}^{-1} \tag{8a}$$

$$\mathrm{k_T} = \left(\frac{\mathrm{V_2}}{\mathrm{V_{LO}}}\right)^2 \cdot \frac{\overline{\mathrm{F}}_{\mathrm{acc}}}{\mathrm{T_{V_2}}} \approx 0.85 \tag{8b}$$

It is stated that k_T is subject to statistical validation, as is C_D at V_2. For the evaluation of this method, C_D/C_L at V_2 was estimated using data taken from Obert [9], which will be described in a following section, and considering $V_2 = 1.2\ V_S$.

3.6 Practical evaluation

For a numerical comparison of these methods, data was gathered from Aircraft manuals, academic books and internet sources of 20 jet-engine aircraft regulated by FAR Part 25 that would serve as inputs and comparison basis. Due to the variety of sources and configurations for the same aircraft model, it was not always possible to cross check the obtained data to assure its accuracy. However, sanity checks and engineering judgement were applied to all of the results. These aircraft can be split as follow:

- 15 airliners (5 wide bodies, 6 narrow bodies and 4 regional jets) and 5 business jets
- 17 twin-engine and 3 tri-engine aircraft
- 11 aircraft with wing mounted engines and 9 with more than one engine at the tail

The diversified types of aircraft used are a good indicator for the representativeness of these results. The procedure used was to select three different weights for each of the aircrafts (including MTOW) and to compare each of the methods estimations with the declared TOFL values. All the evaluations were made considering sea level ISA standard day. From this comparison, the mean of the unsigned relative error, the mean of the signed relative error and the standard deviation of the signed relative are displayed in Table 1. The mean errors allow evaluation of the accuracy of the methods, with the comparison between the signed and unsigned values providing a quick indication of whether a simple linear calibration would improve greatly the accuracy of the results. On the other hand, precision is equally important, and the standard deviation of the signed error is a good indicator of this characteristic.

Table 1: Methods evaluation results

Method	**Unsigned mean error**	**Signed mean error**	**Signed standard deviation**
Roskam	10.5%	-9.7%	6.8%
Kroo	10.4%	9.4%	7.9%
Kundu	13.8%	-13.1%	9.4%
Torenbeek, '82	6.1%	-0.1%	7.2%
Torenbeek, '13	13.2%	-12.6%	8.8%

These results were exhibited in a graphic way as well, in order to improve perception of estimation trends. On the Figure 1 to Figure 5, the shape of the dots indicate engine configuration – triangular means triple-engine, round means twin-engine, wing-mounted, and star means twin-engine, rear-mounted. Visual guidelines were also plotted, with a black line at the bisectrix of the quadrant (where all the point would be placed, for an ideal method) and plus/less 5% deviations from this straight. Also, on the Appendix, Table A1 presents numerically all the data displayed in these figures.

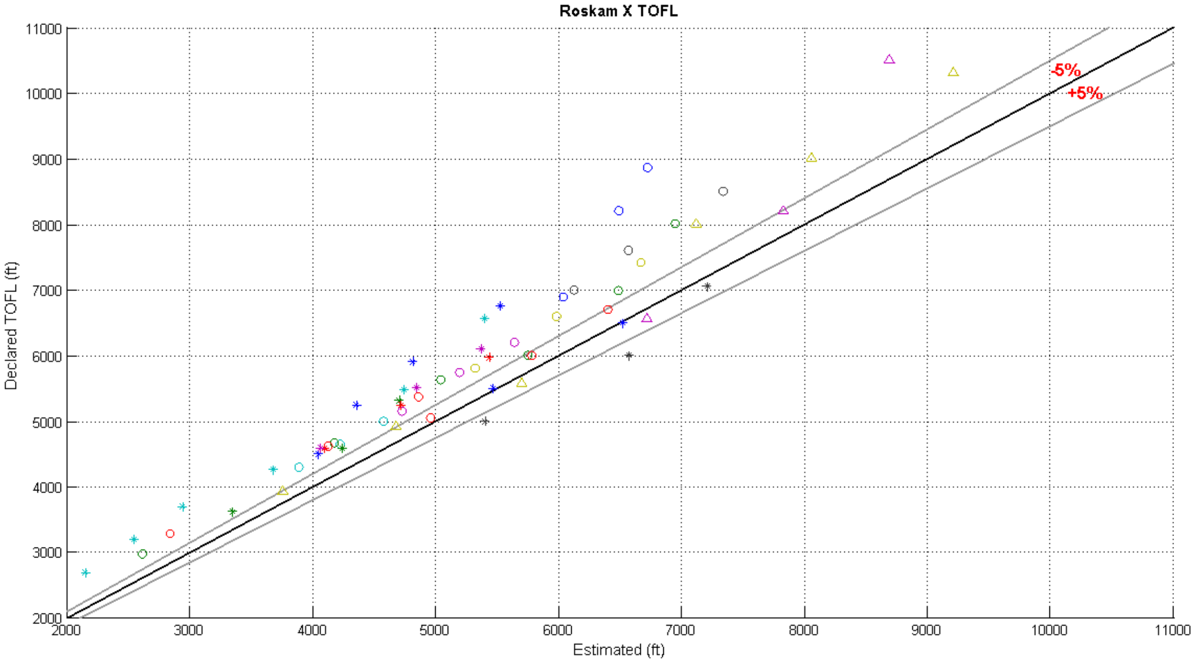

Figure 1: Roskam graphic evaluation

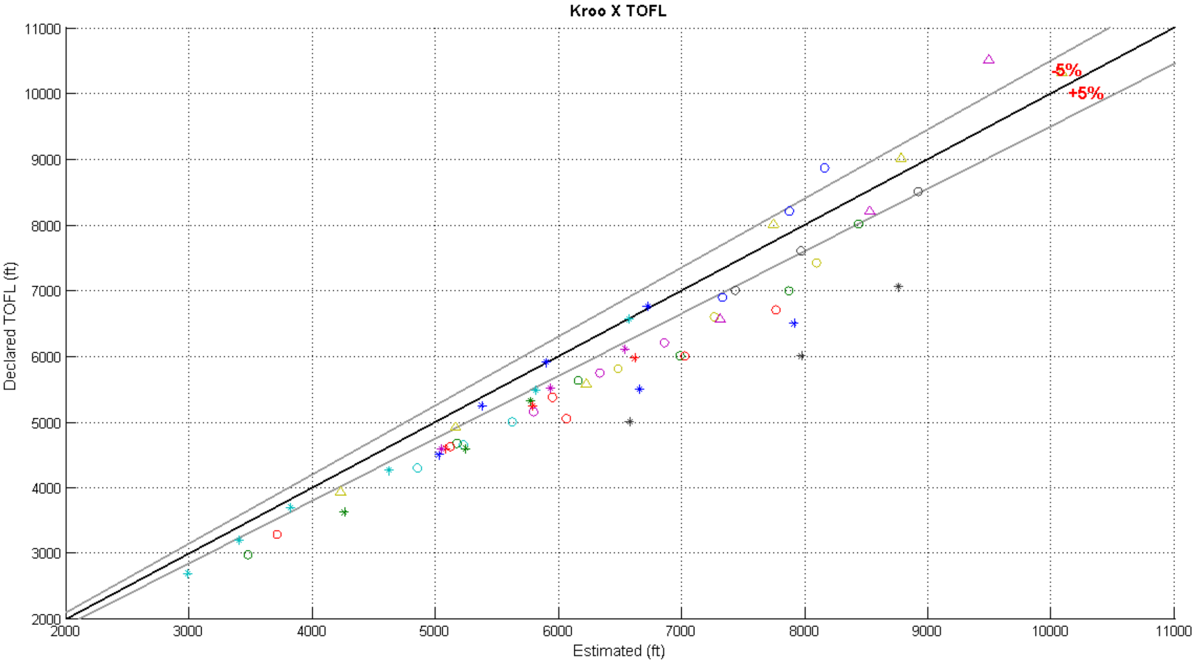

Figure 2: Kroo graphic evaluation

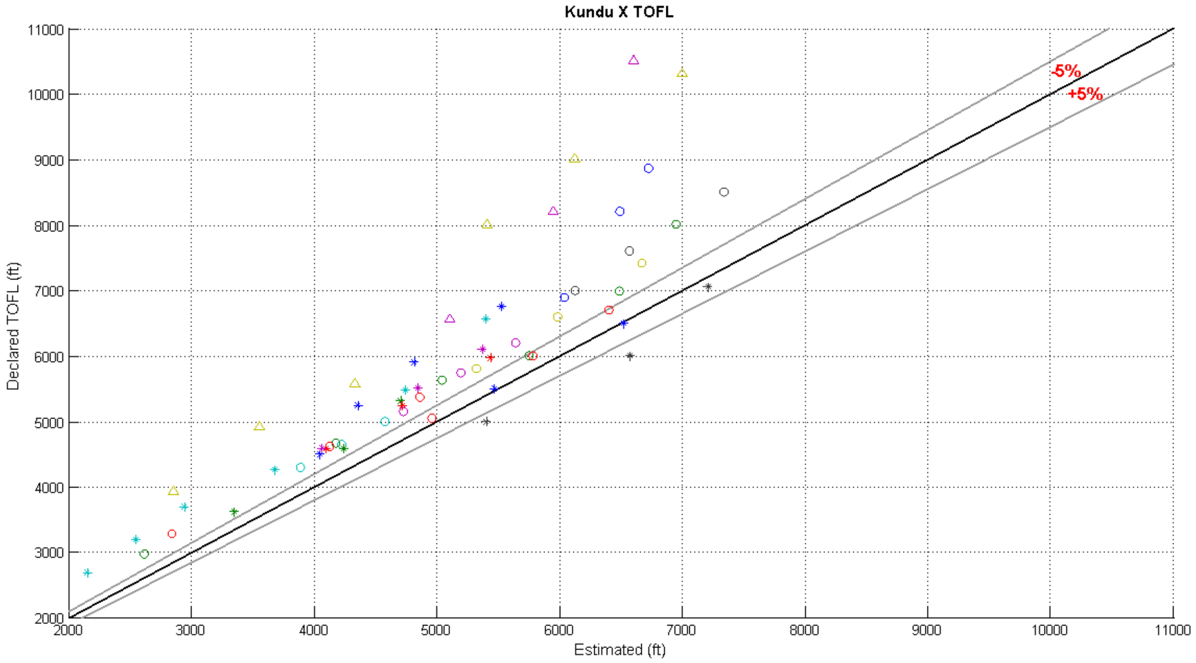

Figure 3: Kundu graphic evaluation

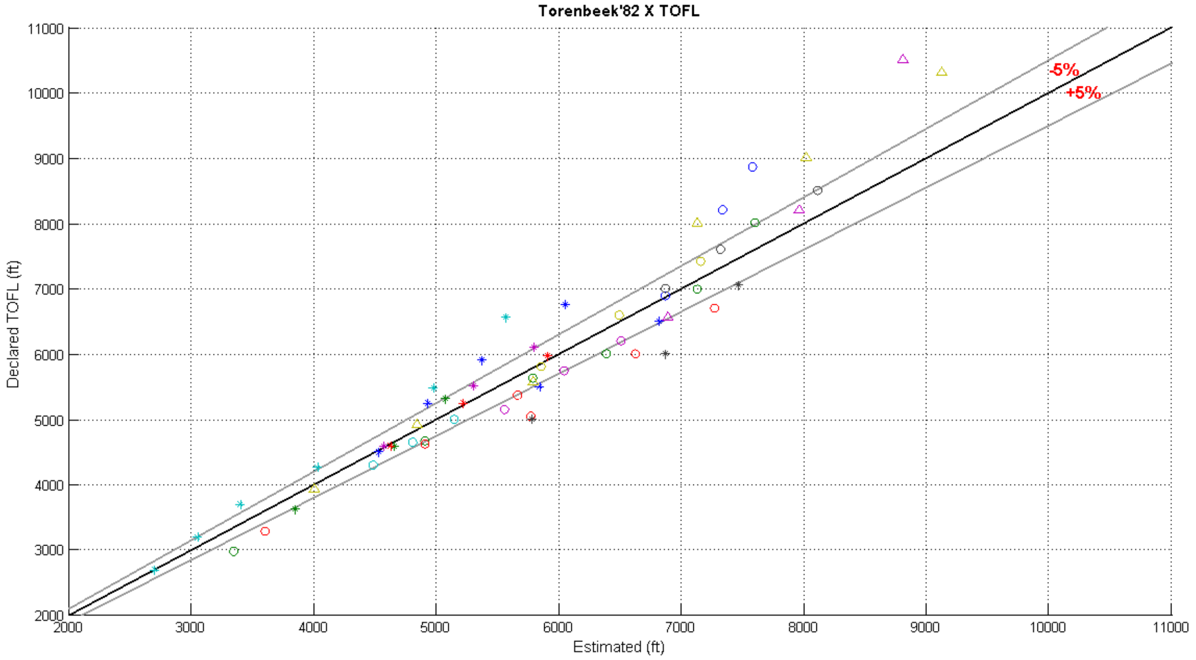

Figure 4: Torenbeek, '82, graphic evaluation

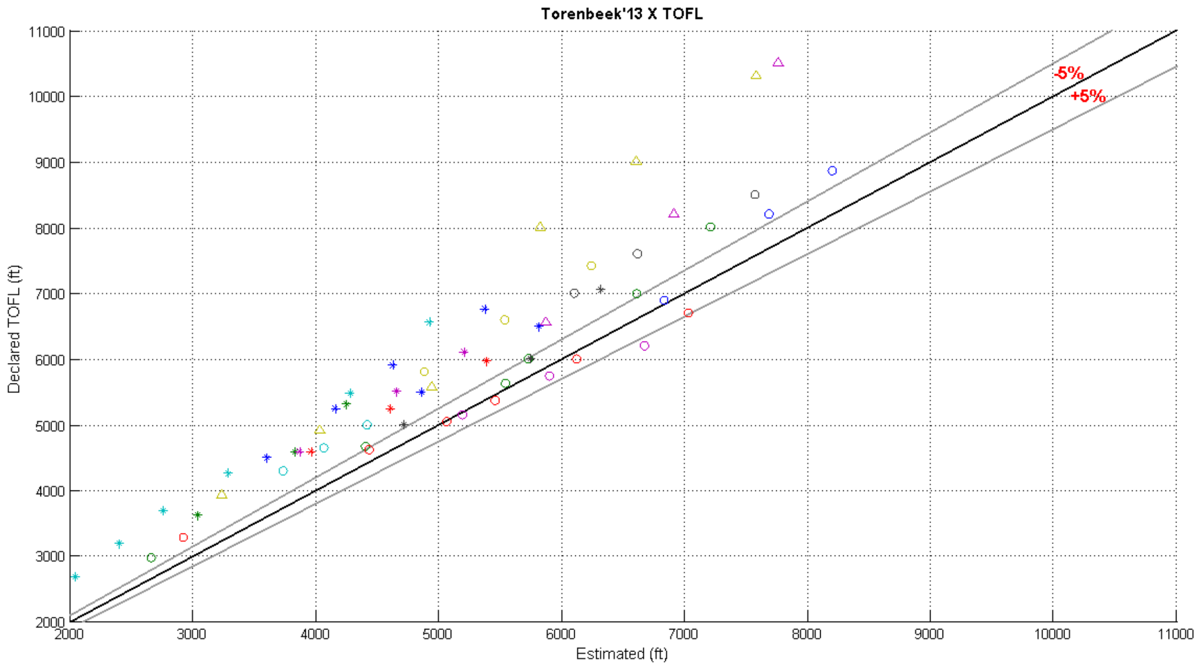

Figure 5: Torenbeek, ’13, graphic evaluation

From the results, it is perceivable that the three first methods present similar results, what would be expected from the similar structures. However, it’s noteworthy the difference of the behavior of the signal of the error, that is inverted, comparing Kroo with Roskam and Kundu. One contributing factor could be the used thrust (average for Kroo and SLS for Roskam and Kundu), indicating that this may be a point of great value. Also catches the eye the better results obtained using Torenbeek, ’82, indicating that the greater complexity of the method is worth it. The value of the signed error is impressive, stating the accuracy of this methodology. The simpler structure and the room for improvement in both precision and accuracy indicate that the results of the other four methods could be improved by a greater margin that Torenbeek, ’82, by the inclusion of new parameters or a new fit for the proposed set of inputs. However, due to the better foundations that it appears to provide, Torenbeek, ’82 [4], was the methodology chosen to be worked upon, in order to greatly increase its capability of estimation.

4 PROPOSED MODIFICATIONS AND METHODOLOGY

To insert modifications on the chosen methodology, taken from Torenbeek, ’82 [4], some auxiliary tools were gathered from alternate references, or developed based on analysis of the results, and will be presented. Next, the assumptions and premises necessary for their integration with the modified method are described, and a mathematical synthesis of the resulting procedure is displayed.

4.1 Obert estimation for lift to drag ratio

At the Low-speed aerodynamics section of his Aerodynamic Design textbook, Obert [9] discusses the usage of high-lift devices in order to improve field performance in transport aircraft. He states that these devices increase drag also, leading to a decrease in the L/D ratio. Based on theoretical studies and real aircraft data, a correlation between C_L at 1.2 V_S, wing aspect ratio, AR, and aircraft L/D is presented. From the plot that was elaborated using lift and drag data from 20 transport jet-engine aircraft, it is possible to propose a linear fit using these parameters:

$$\left(\frac{L}{D}\right)_{1.2\cdot V_S} = 7.262 \cdot \sqrt{AR} - 6.464 \cdot (C_L)_{1.2V_S} \tag{9}$$

Some of the presented methods use L/D estimations in the calculations, and it is by itself a relevant characteristic in the design process, for the second segment climb performance requirements. However, the references in which the TOFL methods are presented do not address this matter. Since the presented data in Obert [9] is grouped in a considerably narrow band in the presented plot, this simple relation may present a good estimation for this meaningful characteristic. The presented data is based on several flap conditions for the selected aircraft, but this should not result in great errors to the estimated result.

4.2 ESDU 76034 for thrust decay with speed

ESDU (originally Engineering Sciences Data Unit) is a collaborative effort that provides validated data and information, mainly in the aeronautical field, aiming to reduce the gap between research and industry. At this specific item [12], it is presented the influence of the ambient conditions, speed and installation effects on the net thrust produced in turbo-jet and turbo-fan engines. For a detailed (from the point of view of aircraft design) set of engine characteristics which may alter the results, several graphs of measured and calculated performance are presented, allowing to estimate via interpolation the thrust variation of a given engine for a chosen speed.

As in the previous case, this information is needed for some of the presented methods, but only Kroo [2] presents a direct way to estimate it, and even more simplified than the one presented in ESDU [12]. Without a good estimation for this characteristic, both engine and aircraft (due to the close relation between required lift and wing area and takeoff speeds) configuration variations are less influential in the final TOFL estimation.

As it is usual for ESDU items, the information is obtained by plotting the inputs in the correspondent figures presented and interpolating the outputs with the adjacent lines. The inputs for this estimation are ambient conditions, engine bypass ratio, engine T_4 (turbine inlet temperature) and evaluated Mach number. Also are necessary engine control method (such as rotation or temperature), engine reference thrust condition and, if applicable, flat rating. To automatize the process, the curves presented by the item were digitalized and numeric interpolation provided the required outputs.

4.3 Engine height influence

Analyzing the results obtained with the presented methods, and mainly the chosen to receive the proposed modifications, it was noted that there was a perceivable separation between aircrafts with rear-mounted and wing-mounted engines, the general direction indicating that estimations for rear-mounted engines were generally more optimistic. Considering the factors that could cause this differentiation in takeoff performance, but were not captured by the methods, the following subjects were raised:

- A rear-mounted engine is usually mounted higher, relatively to the aircraft size, than a wing-mounted engine, which causes a pitch down moment, compromising takeoff run and rotation.
- A wing-mounted engine has a longer arm, from aircraft centerline. This generates a greater yawning moment due to engine failure, and consequently a greater drag in order to compensate this disturbance.
- A rear-mounted engine allows a lower wing height, which could lead to a greater influence of ground effect and improve aerodynamic characteristics.

- A rear-mounted engine is further away from the rotation axis than a wing mounted engine and drives aircraft center of gravity rearwards, which could lead to an increase in rotational inertia and, consequently, an increase in rotation effort.

These effects are very difficult to estimate, even more in early design phases. Also, some of them lead in opposite directions, regarding which configuration should bring better performance, all else being equal. In order to capture the observed separation between the two classes of aircraft, it was chosen a dimensionless parameter comprising engine height to be used as a correction factor, K_{EH}:

$$\mathrm{K_{EH}} = 0.971 + 0.209 \cdot \frac{\mathrm{Z_{eng}}}{\mathrm{S/b}} \cdot \frac{\mathrm{T_{V_2}}}{\mathrm{W_{TO}}} \tag{10}$$

In order to relativize engine height for different aircraft sizes, it was decided to divide engine height for the standard mean chord (wing area divided by span), which can be estimated with greater accuracy than the more usual mean aerodynamic chord. It was included as well the aircraft thrust to weight ratio at $V_{2,}$ since the effects caused by a more powerful engine are understood to be more significant than a smaller one. The coefficients of this parameter were fitted from the database of aircraft used to evaluate the results displayed in section 3.6.

4.4 Assumptions and premises

To integrate the three methodologies already described in section **Erro! Fonte de referência não encontrada.** with the TOFL estimation method of choice (Torenbeek, '82 [4]), some reasonable statements were assumed, regarding both the takeoff procedures and the characteristics of the pool of aircraft used to validate the developed method.

Regarding the takeoff operation, it was assumed that V_2 would always be limited by 1.2 of V_S, which is not necessarily true, despite being the most usual situation. This led to the assumption of C_{L2} equal to 0.694 times the C_{Lmax} for the takeoff configuration. Also, it was assumed that the failed engine would not alter significantly the lift to drag ratio obtained with Obert [9][8], and the second segment climb requirements would be met (these requirements often interfere with takeoff performance, due to the necessity of selecting a different flap setting, for example. However, for sea level ISA conditions, this is usually not a major concern).

The integration of ESDU [12] for thrust estimation required further premises to be assumed. Namely, one of the inputs required for the estimation is T_4, temperature at the turbine inlet. However, this is a restrict information, not trivial to obtain. It has a great influence of the state-of-the-art of the period when the engine was developed, due to the evolution of the materials used for this purpose, as indicated by Heidmann [13]. On his presentation, it is shown a quasi-linear correlation between year and T_4, which was used to estimate this parameter for each of the aircrafts, based on their launch year. With this information, it was assumed that the method of engine control used by each of the engines evaluated was rotation of the low pressure compressor, which could bring non-negligible differences in the results. Finally, it was considered that the engines were not flat-rated, since this information was also not available. In an actual aircraft design, all of these premises could be used as a starting point, and evolve to the real characteristics of the selected engine of the aircraft, as they become clear. With this setup, the aircraft speed was calculated at the desired points, and the available thrust was determined.

Regarding the integration of the engine height correction factor to the chosen TOFL method, it was inserted multiplying the aircraft-dependent term of the equation. This way, the inertia distance remains invariant with the aircraft, and all the remaining is factored by the proposed K_{EH}.

4.5 Mathematical representation

Summarizing the description of the procedures in section 4.4, the following equation is obtained:

$$S_{TOFL} = \frac{0.863}{1+2.3\cdot\Delta\gamma_2}\left(\frac{W_{TO}/S}{\rho\cdot g\cdot C_{L_2}} + h_{TO}\right)\cdot\left(\frac{1}{T_{V_2}/W_{TO}-\mu'} + 2.7\right)\cdot K_{EH} + \frac{\Delta S_{TO}}{\sqrt{\sigma}} \qquad (11a)$$

$$\Delta\gamma_2 = \left(\frac{\left(1-N_{eng}^{-1}\right)\cdot T_{V_2}}{W_{TO}} - \left(\frac{L}{D}\right)_{V_2}^{-1}\right) - \gamma_{2_{req}} \qquad (11b)$$

$$\left(\frac{L}{D}\right)_{V_2} = 7.262\cdot\sqrt{AR} - 6.464\cdot C_{L_2} \qquad (11c)$$

$$C_{L_2} = 0.694\cdot C_{L_{max}TO} \qquad (11d)$$

$$T_{V_2} = \left(K_{ESDU}\right)_{V_2}\cdot T_{SLS} \qquad (11e)$$

$$\mu' = 0.02 + 0.01\cdot C_{L_{max}TO} \qquad (11f)$$

$$K_{EH} = 0.971 + 0.209\cdot\frac{Z_{eng}}{S/b}\cdot\frac{T_{V_2}}{W_{TO}} \qquad (11g)$$

Recapping, ΔS_{TO} corresponds to the inertia distance (655 feet), $\Delta\gamma_2$ corresponds to the excess climb gradient OEI (the second gradient climb that exceeds the minimum required for the specified aircraft, 0.024 to twin-engines and 0.027 to tri-engines for FAR 25), h_{TO} correspond to the screening height that marks the end of takeoff (35 feet for FAR 25) and μ' corresponds to a total deceleration component. K_{ESDU} corresponds to the thrust decay obtained using the method described in 4.2 at V_2.

5 COMPARATIVE RESULTS

Compiling the modifications to Torenbeek, '82, method, the estimation errors were evaluated once again, obtaining the results displayed at Table 2.

Table 2: Modifications evaluation results

Method	Unsigned mean error	Signed mean error	Signed standard deviation
Torenbeek, '82 Unchanged	6.1%	-0.1%	7.2%
Torenbeek, '82 Modified	5.0%	0.1%	6.3%

Despite small, these improvements are significant, given the good level of the results of the original method and the narrow margins involved in this field, as mentioned in section 1 – a 1% TOFL reduction can almost reach the 100 feet example given in that section, indicating the potential gains with a better estimation since early phases. Figure 6 shows, with the same symbol code than the previous plots, the results obtained with the modified method in a visual manner. Table A1 also contains the results for the proposed methodology.

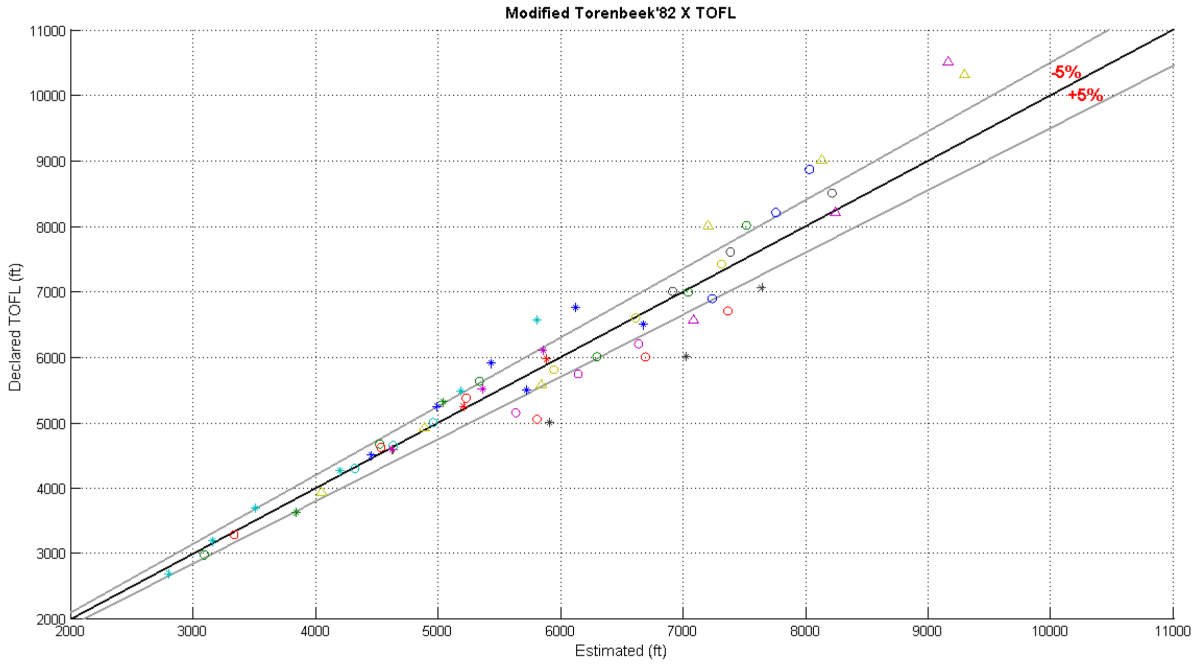

Figure 6: Modified Torenbeek, ’82, graphic evaluation

APPENDIX

Table A1: Reference and estimated TOFL for the selected aircraft, in feets

Aircraft	Ref. Values	Roskam	Kroo	Kundu	Tor'82	Tor'13	Propos. Method
WB 3E WM 1	10499	8681	9485	6598	8801	7755	9155
	8202	7821	8519	5944	7957	6908	8236
	6562	6714	7306	5103	6885	5868	7079
WB 3E WM 2	10302	9202	10079	6993	9118	7576	9286
	9006	8050	8775	6118	8012	6602	8124
	8005	7113	7740	5406	7125	5823	7197
WB 2E WM 1	8858	6719	8153	6719	7577	8194	8023
	8202	6485	7867	6485	7332	7681	7750
	6890	6036	7327	6036	6865	6828	7231
WB 2E WM 2	7415	6665	8087	6665	7153	6239	7306
	6595	5980	7260	5980	6490	5534	6607
	5807	5318	6479	5318	5853	4881	5940
WB 2E WM 3	8497	7333	8912	7333	8106	7567	8208
	7598	6563	7962	6563	7316	6612	7378
	6998	6121	7429	6121	6866	6099	6909
NB 2E WM 1	8005	6944	8429	6944	7595	7205	7509
	6988	6482	7864	6482	7125	6606	7035
	6004	5747	6983	5747	6385	5726	6291
NB 2E WM 2	6700	6396	7759	6396	7268	7025	7359
	6000	5780	7021	5780	6622	6119	6687
	5050	4956	6059	4956	5769	5064	5803
NB 2E WM 3	5000	4573	5621	4573	5145	4417	4958
	4650	4223	5226	4223	4807	4066	4631
	4300	3888	4852	3888	4484	3734	4319
NB 2E WM 4	6200	5639	6855	5639	6504	6670	6630
	5740	5193	6333	5193	6038	5898	6139
	5151	4725	5794	4725	5554	5191	5630
NB 2E TM 1	6562	5398	6572	5398	5566	4924	5805
	5479	4743	5814	4743	4978	4281	5186
	4265	3680	4623	3680	4032	3291	4197
NB 2E TM 2	7054	7203	8750	7203	7461	6316	7635
	6004	6567	7967	6567	6865	5745	7020
	5000	5400	6574	5400	5780	4717	5907
RJ 2E WM 1	5370	4859	5947	4859	5660	5456	5225
	4623	4127	5118	4127	4906	4433	4530
	3291	2841	3716	2841	3604	2925	3335
RJ 2E WM 2	5630	5040	6156	5040	5785	5540	5335
	4672	4176	5174	4176	4905	4403	4521
	2982	2617	3479	2617	3347	2665	3092
RJ 2E TM 1	6759	5523	6719	5523	6051	5375	6116
	5906	4814	5896	4814	5368	4627	5429
	5249	4355	5374	4355	4929	4162	4988
RJ 2E TM 2	6496	6518	7907	6518	6819	5812	6671
	5499	5467	6652	5467	5842	4859	5720
	4501	4046	5028	4046	4532	3604	4452

BJ TM 3E 1	5577	5697	6221	4330	5789	4941	5839
	4921	4676	5161	3554	4842	4032	4886
	3937	3756	4230	2854	4005	3236	4047
BJ TM 2E 1	3700	2944	3825	2944	3403	2760	3511
	3200	2551	3408	2551	3054	2404	3160
	2700	2154	2994	2154	2702	2047	2806
BJ TM 2E 2	5971	5435	6615	5435	5904	5385	5884
	5249	4719	5787	4719	5218	4607	5205
	4593	4097	5085	4097	4626	3965	4622
BJ TM 2E 3	5315	4703	5769	4703	5075	4250	5039
	4593	4239	5244	4239	4655	3832	4628
	3629	3349	4262	3349	3849	3042	3843
BJ TM 2E 4	6102	5369	6538	5369	5797	5204	5853
	5512	4846	5932	4846	5304	4656	5358
	4593	4064	5048	4064	4572	3871	4626

Notes:

- WB – Wide body, NB – Narrow body, RJ – Regional jet, BJ – Business jet
- 2E – Twin-engine, 3E – Tri-engine
- WM – Wing mounted engines, TM – Tail mounted engines

REFERENCES

[1] ROSKAM, Jan. Airplane Design, Part I: Preliminary Sizing of Airplanes. Ottawa: Roskam Aviation and Engineering Corporation, 1985. Section 3.2.3.

[2] KROO, Ilan. Aircraft Design: Synthesis and Analysis. Stanford: Desktop Aeronautics, 2001. Section Takeoff.

[3] KUNDU, Ajoy Kumar. Aircraft Design. Cambridge: Cambridge University Press, 2010. Section 11.3.1.

[4] TORENBEEK, Egbert. Synthesis of Subsonic Airplane Design. Delft: Delft University Press, 1982. Section 5.4.5.

[5] TORENBEEK, Egbert. Advanced Aircraft Design. 2013. Section 9.4.4.

[6] LOFTIN, Jr., Laurence K. Subsonic Aircraft: Evolution and the Matching of Size to Performance. NASA Reference Publication 1060, Hampton, 1980. Section 3.4.3.

[7] RAYMER, Daniel P. Aircraft Design: A Conceptual Approach. 2 ed. Washington: AIAA, 1992. Sections 5.3 and 17.8.

[8] ROSKAM, Jan. Airplane Design, Part VII: Determination of Stability, Control and Performance Characteristics: FAR and Military Requirements. Ottawa: Roskam Aviation and Engineering Corporation, 1988. Section 5.2

[9] OBERT, Ed. Aerodynamic Design of Transport Aircraft. Delft: IOS Press, 2009. Chapter 27.

[10] ESDU, First approximation to take-off field length of multi-engined transport aeroplanes. Performance Series, Section 14: Estimation - Take-off, Item No. 76011, Amendment A. Engineering Sciences Data Unit, May 1985.

[11] ESDU, Calculation of ground performance in take-off and landing. Performance Series, Section 11: Airfield Performance - General, Item No. 85029. Engineering Sciences Data Unit, November 1985.

[12] ESDU, Estimation of take-off thrust using generalized data for turbo-jet and turbo-fan engines. Performance Series, Section 7: Estimation - Jet and Fan Engine Thrust, Item No. 76034. Engineering Sciences Data Unit, November 1976.

[13] HEIDMANN, James. Improving Engine Efficiency Through Core Developments. Presentation at AIAA Aero Sciences Meeting: January 2011.