Conception of Sub-Optimal Solution for Spacecraft Rendezvous Near an Elliptic Orbit

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Abstract This document presents a part of work which aims to find sub-optimal strategy for the orbital rendezvous between an active chaser spacecraft and a passive target satellite which is moving in a known elliptic orbit around the Earth. The Yamanaka-Ankersen model of motion is considered. The variable-mass chaser spacecraft is equipped with a variable-thrust propulsion. The essence of the problem is to find a control resulting in a quasi-optimal rendezvous trajectory. This work approaches the problem of rendezvous of spacecraft using model predictive control. A proposal of solution is based on a version of Quasi Time-Optimal Receding Horizon Control (QTO-RHC) algorithm. This method is noise resistant and able to effectively handle with various constraints. The problem includes constraints on amount of used fuel, thrust magnitude and approach velocity. In this paper a conception of solution is presented. The paper contains also results for simplified case.

1 Introduction

The rendezvous maneuver is accomplished when both satellites attain the same position and velocity, both vectors, at the same time. In this investigation the orbital rendezvous maneuver is considered for the case when the active chaser spacecraft has engines which can impart variable thrusts independently in three perpendicular directions and the target satellite (nonmaneuvering) is moving in a known elliptic

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orbit. The distance between the two vehicles is small compared to the radius of the target vehicle orbit.

The study of relative motion between satellites such as rendezvous maneuvers is of great importance in space technology. Until today all space missions involving close proximity maneuvers has been for rendezvous missions on circular orbits. This is going to change for future planned, more complex missions where target object is moving in elliptic orbit.

The relative dynamics between two spacecraft or bodies has been researched by several in the past. The most well known and used relative motion equations were derived by Clohessy and Wiltshire in 1960 [4]. These equations were obtained by studying the Hill's model from 1878 [6]. This model is applied for the case when the target vehicle is moving in a circular orbit. Works on elliptic orbits have been less addressed in the literature. Tschauner and Hempel obtained complex homogeneous solution for elliptic orbits in 1965 [9]. Present investigation is based on Yamanaka-Ankersen equations obtained in 2002 [10].

This work is a conception of solution for the problem of rendezvous of spacecraft using model predictive control. The predictive control algorithm is based on version of Quasi Time-Optimal Receding Horizon Control (QTO-RHC) algorithm presented in Bania PhD thesis [2]. The algorithm allows a realisation of time-optimal control tasks and stabilization after reaching target.

2 Model of Relative Motion

In this investigation it is assumed that the motion is under the action of a central gravity field and forces from thruster actuation or disturbances. The distance between the vehicles is small compared to the radius of the target orbit.

2.1 Reference Frame

The approach trajectory of the chaser is described in the local orbital frame of the target. This frame is often referred to as the local-vertical/local-horizontal (LVLH) frame. For the analysis of rendezvous trajectories, it is best to use a reference frame originating in the center of mass of the target vehicle, i.e. to look at the chaser motion as an astronaut sitting in the target vehicle would. The situation is shown in Fig. 1.

- Axis $\mathbf{X}_1 : \mathbf{X}_1 = \mathbf{X}_2 \times \mathbf{X}_3$ ($\mathbf{X}_1$ is in the direction of the orbital velocity vector but not necessarily aligned with it). In the rendezvous literature this coordinate is also called **V-bar**,
- axis $\mathbf{X}_2$: in the opposite direction of the angular momentum vector of the orbit. In the rendezvous literature this coordinate is also called **H-bar**,
- axis $\mathbf{X}_3$: radial from the spacecraft CoM to the centre of the Earth. In the rendezvous literature this coordinate is also called **R-bar** [5].

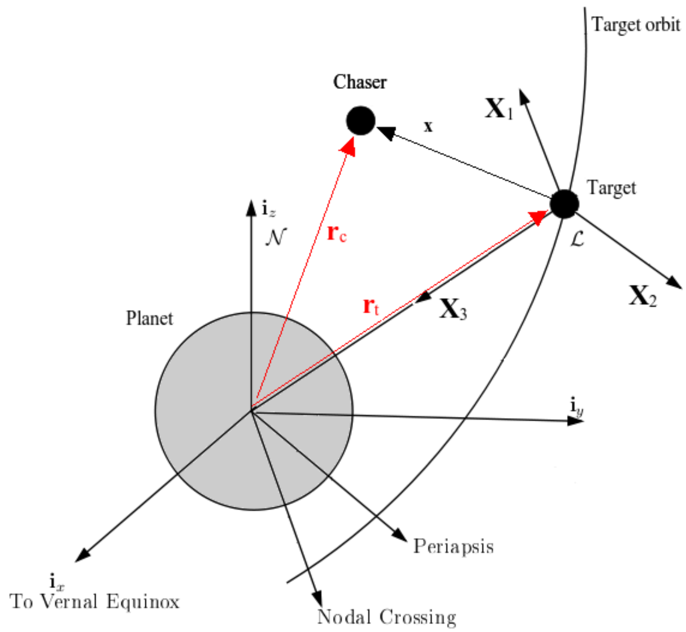

Fig. 1 Reference frame [7]

2.2 Equations of Motion

The equations of motion of the chaser relative to the target vehicle are shown below [1]. Equations can be applied for the case when target vehicle is moving in ellliptic orbit.

$$\ddot{x}_1 - \omega^2 x_1 - 2\omega\dot{x}_3 - \dot{\omega}x_3 + \kappa\omega^{\frac{3}{2}}x_1 = \frac{F_1}{m_c}, \tag{1}$$

$$\ddot{x}_2 + \kappa\omega^{\frac{3}{2}}x_2 = \frac{F_2}{m_c}, \tag{2}$$

$$\ddot{x}_3 - \omega^2 x_3 + 2\omega\dot{x}_1 + \dot{\omega}x_1 - 2\kappa\omega^{\frac{3}{2}}x_3 = \frac{F_3}{m_c}, \tag{3}$$

where:

$$\omega = \left| \frac{\mathbf{r}_t \times \mathbf{v}_t}{|\ \mathbf{r}_t\ |^2} \right| \tag{4}$$

$$\kappa = \mu \left(\frac{1}{h}\right)^{\frac{3}{2}}, \tag{5}$$

$$\mu = GM, \tag{6}$$

$$h = \frac{L}{m_t} = r_t^2 \omega, \tag{7}$$

x_i for $i = 1,2,3$ - chaser position vector components (relative to target vehicle),
ω - target vehicle angular velocity vector magnitude (a function of time),
$\mathbf{r}_t$, $\mathbf{v}_t$ - target vehicle position and velocity vector respectively (relative to the centre of the Earth, see Fig. 1),
κ - constant for elliptical orbits,
μ - gravitational parameter,
G - universal gravitational constant,
M - Earth mass,
h - normalized angular momentum vector magnitude,
L - angular momentum vector magnitude,
m_t - target vehicle mass,
r_t - target vehicle position vector magnitude (relative to the centre of the Earth),
F_i for $i = 1,2,3$ - thrust force vector components,
m_c - chaser vehicle mass.

Following equations (8), (9) and (10) are the special case of equations (1), (2) and (3). There are referred as Hill-Clohessy-Wiltshire equations and they are valid for near circular and circular orbits only [1].

$$\ddot{x}_1 - 2\omega\dot{x}_3 = \frac{F_1}{m_c}, \tag{8}$$

$$\ddot{x}_2 + \omega^2 x_2 = \frac{F_2}{m_c}, \tag{9}$$

$$\ddot{x}_3 - 3\omega^2 x_3 + 2\omega\dot{x}_1 = \frac{F_3}{m_c}. \tag{10}$$

3 Model Predictive Control Formulation

The essence of the problem is to find a control resulting in a quasi-optimal rendezvous trajectory for system described by equations (1), (2) and (3). The solution should satisfy constraints on amount of used fuel, thrust magnitude and approach velocity. This investigation uses model predictive control to solve the problem. Objective function allows to choose between four objectives during the process.

3.1 Constraints on the Problem and Boundary Conditions

Rendezvous maneuver requires to satisfy the following boundary conditions:

$$\mathbf{r}_c\left(t_f\right) = \mathbf{r}_t\left(t_f\right), \tag{11}$$

$$\mathbf{v}_c\left(t_f\right) = \mathbf{v}_t\left(t_f\right), \tag{12}$$

where:
t_f - the end of the rendezvous maneuver,
$\mathbf{r}_c$ - chaser vehicle position vector (relative to the centre of the Earth),
$\mathbf{v}_c$ - chaser vehicle velocity vector (relative to the centre of the Earth).
Control inputs are bounded as follows:

$$-u_{i_{max}} \le u_i \le u_{i_{max}} \tag{13}$$

where $u_{i_{max}}$ is the maximum value of control signal component.
Constraint on amount of used fuel:

$$\varepsilon_f \int_{t_0}^{t_f} \mid \mathbf{u}(t) \mid dt \le m_f(t_0) \tag{14}$$

where:
ε_f - weighting factor,
t_0 - the beginning of the rendezvous maneuver ($t = 0$),
$\mathbf{u}(t)$ - control vector,
m_f - fuel mass.
Constraint on approach velocity is included in cost function ($v_{req}(\mid \mathbf{x} \mid)$).

3.2 Objective Function

Following cost function is applied:

$$Q(\mathbf{u},T) = \lambda_1 T + \lambda_2 \int_0^T \mid \mathbf{u}(t) \mid^2 dt + \lambda_3 \mid \mathbf{x}(T) \mid^2 + \lambda_4 \mid\mid \dot{\mathbf{x}}(T) \mid - v_{req}(\mid \mathbf{x} \mid) \mid^2 \tag{15}$$

where:
T - prediction horizon,
λ_1, λ_2, λ_3, λ_4 - weighting factors,
$v_{req}(\mid \mathbf{x} \mid)$ - required relative velocity in proximity of target object. This is an implementation of constraint on approach deceleration and enforces desirable rate of velocity decrease.

$$v_{req}(\mid \mathbf{x} \mid) \sim \mid \mathbf{x} \mid. \tag{16}$$

Such implementation of objective function allows to choose between minimization of the time of the maneuver (factor λ_1), minimization of fuel expenditure (λ_2), stabilization in proximity of target (λ_3) and enforcing desired approach deceleration (λ_4). The factors are adjusted during the process.

4 Predictive Control Algorithm Conception

The conception of predictive control algorithm is based on version of Quasi Time-Optimal Receding Horizon Control (QTO-RHC) algorithm presented in Bania PhD thesis [2]. Algorithm is resistant to relatively large noise of the control signal as well as state estimation error and is able to handle with noise greater than it occurs in space technology. Algorithm scheme is presented in Fig. 2:

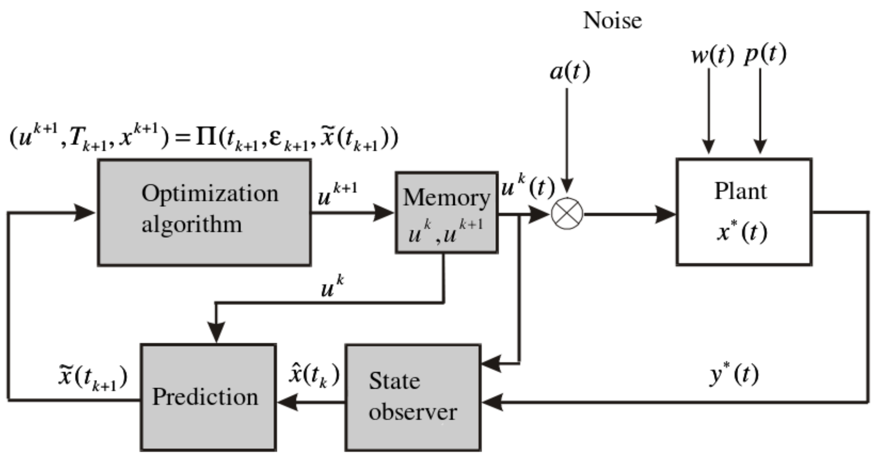

Fig. 2 Predictive control algorithm scheme [2]

The QTO-RHC algorithm uses MSE (*Monotone Structural Evolution*) method for solving the optimal control problem. It is effective for a large class of nonlinear problems with control and state constraints, including singular cases. MSE originates from the *variable parameterization* method. A distinctive feature of MSE is that the decision space is systematically reconstructed in the course of optimization, with changing the control structure, parameterization and, typically, the number of decision variables. The search for structural changes which lead to rapid improvement of the performance index is based on analysis of the discrepancy between the current approximation of solution and the maximum principle optimality conditions, and continues until these conditions are satisfied with sufficient accuracy. The proper choice of the sequence of decision spaces, utilizing information taken from the adjoint solution, allows the number of decision variables to be kept comparatively small, at least in early stages of optimization. In consequence, quasi-Newton or Newton optimization with analytical gradients can be used, which is vital for fast convergence. The dimension of the decision space grows only when this is necessary for improving accuracy of optimal control approximation. An important property of MSE is that the performance index decreases monotonously during optimization, due to control preservation by the structure changes [8].

5 Preliminary Calculations Using Simplified Case

A preliminary calculations were conducted using model described by equations (8), (9) and (10). Using these equations assume state equations:

$$\dot{x}_1 = x_4, \tag{17}$$

$$\dot{x}_2 = x_5, \tag{18}$$

$$\dot{x}_3 = x_6, \tag{19}$$

$$\dot{x}_4 = \ddot{x}_1 = 2\omega\dot{x}_3 + \frac{F_1}{m_c}, \tag{20}$$

$$\dot{x}_5 = \ddot{x}_2 = -\omega^2 x_2 + \frac{F_2}{m_c}, \tag{21}$$

$$\dot{x}_6 = \ddot{x}_3 = 3\omega^2 x_3 - 2\omega\dot{x}_1 + \frac{F_3}{m_c}. \tag{22}$$

Using matrix notation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2\omega \\ 0 & -\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3\omega^2 & -2\omega & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_c} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_c} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{23}$$

can be written in the form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}. \tag{24}$$

Such model was introduced into Matlab MPC toolbox software. In this simplified calculation it was assumed that the chaser has constant mass equal to 500 kg and the target vehicle is moving in stationary orbit.

Initial conditions:

$$\mathbf{x}(t_0) = \begin{bmatrix} -10000 \\ -10000 \\ -10000 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (25)$$

$$\mathbf{u}(t_0) = \mathbf{0}. \quad (26)$$

Constraint on i-th control vector component $i = 4,5,6$ (N):

$$-1000 \leq u_i \leq 1000. \quad (27)$$

Constraint on i-th control rate vector component $i = 4,5,6$ N/s:

$$-100 \leq \dot{u}_{4,5,6} \leq 100. \quad (28)$$

Constraint on final state $i = 1,2,3$ (m):

$$-0.5 \leq x_i\left(t_f\right) \leq 0.5, \quad (29)$$

for $i = 4,5,6$ (m/s):

$$-0.01 \leq x_i\left(t_f\right) \leq 0.01. \quad (30)$$

The controller will try to minimize the deviation of each output from its setpoint or reference value. For each sampling instant in the prediction horizon, the controller multiplies predicted deviations for each output by the output's weight, squares the result, and sums over all sampling instants and all outputs. One of the controller's objectives is to minimize this sum [3]. The output weights for x_1, x_2, x_3 are 1, and for x_4, x_5, x_6 are 0.5.

Control interval is equal to 1 s , prediction horizon is equal to 200 intervals and control horizon 10 intervals. Time of simulation was set to 1800 s.

The results of simulation are presented in Fig. 3 and Fig. 4.

Fig. 3 and Fig. 4 shows that system achieved constraints on state and control in 1200 s.

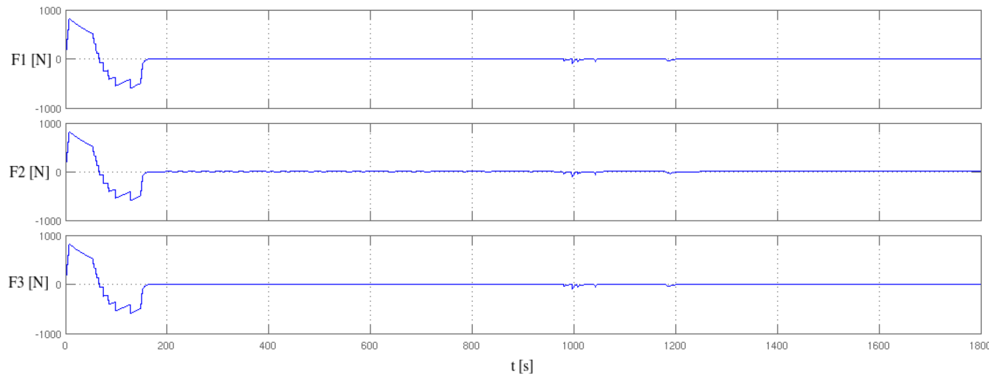

Fig. 3 Control history

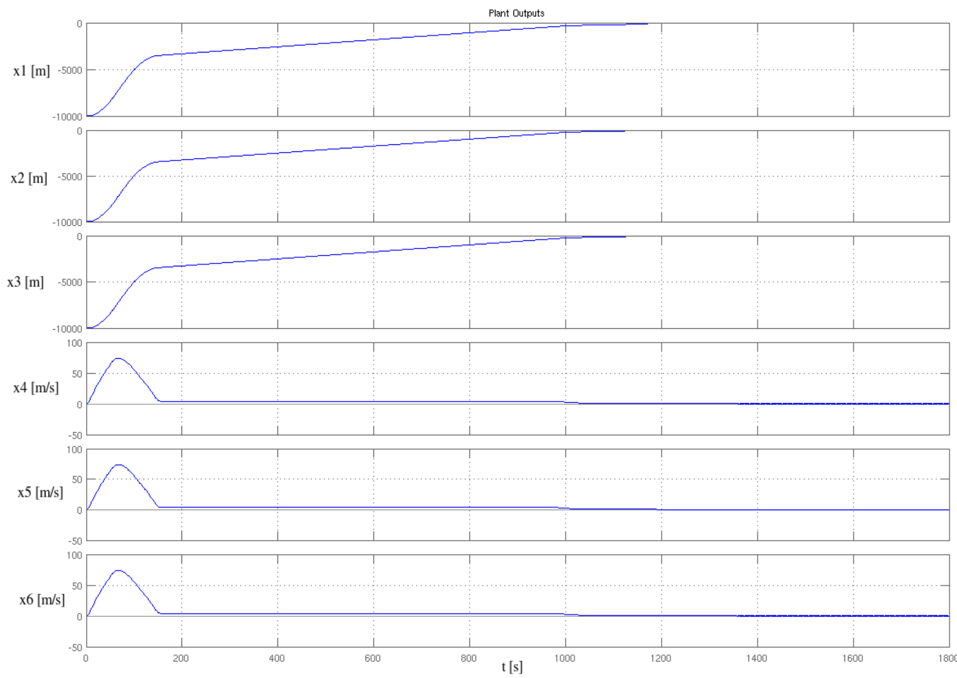

Fig. 4 State trajectory

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