



# Thermally Induced Vibration Analysis of Flexible Solar Wing

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#### ABSTRACT

Flexible solar wings of large scale have been used to provide electricity for spacecrafts. When the solar wings running on orbits with spacecrafts, they suffer from periodically thermal loads. And the thermal loads will cause thermal deformations of the solar wings, even thermal vibrations. Finite element method and equivalent displacement method were utilized to investigate the thermally induced vibration of flexible solar wing. Some Python programs were written to implement pre- and post-processing for finite element analysis. Dynamic responses of a flexible solar wing under thermal loads were analyzed, and the effect of the thermal loads on the flexible solar wing were illustrated by the displacement responses of some points of the flexible solar wing. The rigid-flexible coupling phenomenon of flexible solar wing was found by the displacement responses. The thermally induced vibration analysis with finite element method was accurate and efficient.

#### NOMENCLATURE

ρ	= density, kg/m <sup>3</sup>
$C_p$	= specific heat capacity at constant pressure, $J/(K \cdot m3)$
$\dot{T}$	= temperature field, K
t	= time, s
k	= thermal conductivity, $W/(m \cdot K)$
<i>x, y, z</i>	= coordinates, m
$q_v$	= heat source, $W/(m^3 \cdot s)$
$\mathcal{E}_{x}, \mathcal{E}_{y}, \mathcal{E}_{z}$	= direct strains
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	= shear strains
$\Delta T$	= increment of temperature, K
α	= coefficient of thermal expansion, $1/K$
М	= mass matrix of finite element model
С	= damping matrix
K	= stiffness matrix
u, ü, ü	= vectors of displacement, velocity and acceleration
F	= vector of loads





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### **1** INTRODUCTION

Flexible solar wing is the main power equipment of space station. Its reliability decides whether the space station runs normally and safely. The flexible solar wing is subjected to several thermal loads such as solar radiation during running on the Earth orbit with space station. The temperatures of different parts of solar wing are different at the same time and vary during an orbital period. Thermal deformation of solar wing occurs periodically, even thermally induced vibration maybe occur. The most notable problem of thermally induced vibration is that of the flexible rolled-up solar array of Hubble Space Telescope, and it was studied by Thornton et al[1] with uncoupled and coupled thermal-structural analyses. Thermally induced vibration of a solar array due to self-shadowing of the central truss was also studied by Thornton et al[2] considering cross-member shadowing and parallel-member shadowing of the truss longerons. Finite element method was used to analyze thermally induced vibrations of the solar array of Hubble Space Telescope by Yao et al[3]. The torsional vibration of the Hubble solar array was studied Huang et al[4]. Thermally induced vibration of the main mast of a solar array was also studied using finite element method by An Xiang et al<sup>[5]</sup> and a method of coupled thermal-structural analysis in whole time domain and uncoupled thermal-structural analysis in sub time step was presented. Uncoupled thermal-structural dynamic analysis method was used to study the thermally induced vibration of deployable antenna by Liu et al[6], and coupled thermal-structural dynamic analysis method was used to study the thermally induced vibrations of flexible beams by Shen et al[7].

Both coupled and uncoupled thermal-structural analysis methods are widely used on the researches of thermally induced vibrations of spacecrafts. The uncoupled method assumes the temperature field is not affected by the displacement field, thus the time-dependent (transient) temperature field of spacecraft can be calculated independently[8-10]. And the temperature field can be used for thermal stress analysis, thermal deformation analysis and uncoupled thermally induced vibration analysis. An analysis approach of uncoupled thermally induced vibration was developed based on equivalent displacement method by Kong et al[11-14], and this approach was also used to study the dynamic response of a flexible solar wing subjected to several kinds of thermal loads under space environment in this paper.

#### 2 METHODOLOGY AND MODELING

# 2.1 Basic Theories and formulas

The governing equation of transient temperature field can be written as

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial y^2} + k \frac{\partial^2 T}{\partial z^2} + q_v$$
(1)

Thermal strains occur when the temperature of an elastic body varies. As isotropic elastic body, the relationship of thermal strains and increment of temperature can be written as

$$\mathcal{E}_x = \mathcal{E}_y = \mathcal{E}_z = a\Delta T, \quad \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$$
 (2)

Thermal stresses usually occur along with thermal strains by the effect of boundary conditions or the interaction between different parts of the elastic body. Thermal deformation and displacement usually occur at the same time. The equation of motion of finite element model is written as

$$M\ddot{\boldsymbol{u}}(t) + C\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t)$$
(3)

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Thermally induced vibration can be expressed by Eq.(3) with F(t) replaced by thermal loads. Equivalent displacement method can be used to transform the thermal loads (such as temperature) to mechanical loads. The relationship of displacements and mechanical loads can be written as

$$Ku = F$$

(4)

For a finite element model, if the displacements caused by a vector of mechanical loads equal to that caused by thermal loads, then the vector of mechanical loads can be used to calculate the thermally induced vibration with Eq.(3). The mechanical loads can be called as equivalent thermal loads, and the method to calculate the equivalent thermal loads is named as equivalent displacement method.

# 2.2 Finite Element Model of Flexible Solar Wing

A flexible solar wing contains three main parts: mast, solar array and storage box, shown as Figure 1. It is about 25 meters long and 6 meters wide. The thickness of the solar array is no more than 1 millimeter. The storage boxes and the mast are connected by multiple-point constraints of beam type (MPC-Beam), and the solar arrays and the storage boxes are connected by multiple-point constraints of pin type (MPC-Pin). The left end of mast is fixed.



Figure 1: Finite element model of flexible solar wing

The detail of the mast of flexible solar wing is shown in Figure 2. There are 3 longitudinal beams with the serial numbers of L1, L2 and L3, and 3 transverse beams of each section of the mast with the serial numbers of T1, T2 and T3. The temperatures of the members of the mast are different in an orbital period, and the temperatures of 4 parts of the mast are shown in Figure 3 where X1 is the diagonal truss at the same side of T1.









#### 3 RESULTS AND ANALYSIS

#### 3.1 Thermally Induced Vibration of Mast

Thermally induced vibration of mast was calculated separately in order to compare the result with that calculated with the whole flexible solar wing.

The response of mast under thermal loads was studied using 3 analysis methods: quasi-static analysis, one-step analysis and two-step analysis, and the results of the 3 method are shown in Figure 4. The results of quasi-static analysis are the curves of which the labels are U1-T, U2-T and U3-T, and U1, U2 and U3 are the displacements on X, Y and Z axes. The curves with 1S and 2S in the labels are the results of one-step analysis and two-step analysis respectively. The initial conditions of dynamic analyses is different between one-step and two-step methods. The initial displacement vector u(t=0) of one-step method is zero, and the equivalent thermal load F(t) is used to calculate the dynamic response of mast. While two-step method consists of a static step and a dynamic step, the static step is used to calculate the displacement u(t=0) with the load  $F(t=0) \neq 0$ , thus the initial displacement of dynamic step is not zero. For one-step analysis, the impact effect caused by F(t) is serious, and obvious vibration occurs at the beginning of the dynamic analysis of the mast. While for two-step analysis, the initial displacement of dynamic step is caused by  $F(t=0) \neq 0$ , and F(t) varies slowly, thus the impact effect is very small and there is no obvious vibration at the beginning of the dynamic analysis of the mast.

Quasi-static displacement responses of one orbital period and dynamic displacement responses of four orbital periods are shown in Figure 4. The curves of the 3 method are very similar, and it means that there is no obvious vibration of the mast during an orbital period. There are obvious vibrations in the initial period of one-step method, but it is not the true result, because this phenomenon is caused by the initial conditions, and it can be seen that there is no obvious vibration in the following orbital periods. The simulation results illustrate that the status of the mast on orbit is quasi-statically thermal deformation, and thermally induced vibration of the mast dose not occur during orbital periods.







Figure 4: Displacement Response of Point P1 of Mast

# 3.2 Thermally Induced Vibration of Flexible Solar Wing

The displacement responses of thermally induced vibration of flexible solar wing are shown in Figure 5 where U1, U2 and U3 are the displacement responses on X, Y and Z axes respectively. P1 is the point at





the right end of the mast, and P2 is the central point of an array. The positions of P1 and P2 are shown in Figure 1.



Figure 5: Displacement Response of Point P1 of Mast and P2 of Array

The time histories are 2 orbital periods shown in Figure 5(a) and Figure 5(b), 4 orbital periods in Figure 5(c) and 8 orbital periods in Figure 5(d). The displacement responses of point P1 of mast shown in Figure 5 are similar with that shown in Figure 4 excepting some vibrations of small amplitudes and high frequencies in Figure 5. There is no obvious vibration of the point P2 of array on X and Y axes, but an





obvious vibration of point P2 occurs on Z axis. Compared with Figure 4, it can be inferred that the vibrations of mast and solar wing are caused by the interaction of the rigid mast and flexible solar wing.

#### 4 CONCLUSION

The results of thermally induced vibration analysis of flexible solar wing illustrate that periodically thermal deformations of rigid mast can cause vibrations of flexible solar wing, and vibrations of small amplitudes and high frequencies of mast can be caused by the vibrations of flexible solar wing. The equivalent displacement method is efficient and accurate to transform thermal loads to mechanical loads.

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