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ON THE MINIMIZATION OF CRUISE DRAG DUE TO PITCH TRIM

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ABSTRACT

A blended-wing-body is an example of an aircraft configuration with multiple control surfaces. The most effective use of these control surfaces, e.g. to minimize cruise drag due to pitch trim, or to maximize pitching moment at low speed in an engine-out condition, leads to optimization problems. The method investigated is this paper is applied to multiple control surfaces, taking into account their mutual interactions and also the influence of shifts of center of gravity. It is shown in a particular case that it is possible to achieve pitch trim in cruise with drag reduction relative to the untrimmed case.

1 INTRODUCTION

Minimizing drag in cruise is most important for aircraft emissions and economics since this is the longest phase of most flights, and where a large proportion of the total fuel is burnt. It is therefore desirable to achieve pitch trim at cruise with the smallest possible drag penalty relative to the untrimmed aircraft; even better would be to achieve pitch trim in cruise with a drag reduction relative to the untrimmed aircraft. There is a limited choice of control surfaces in the traditional aircraft configuration, with a fuselage to carry payload, a wing to carry fuel and provide lift and roll control, and an empenage to provide yaw and pitch control. In a blended wing body (BWB) configuration, with control surfaces both on the centerbody and wing, there are multiple control surfaces; thus arises the question of which is the best combination of all available control surfaces to achieve pitch trim with least drag.

The method to be presented applies to the trimming of an aircraft, for any axis and any flight phase, when there is a choice of control surfaces to be used. It allows distinct control surfaces to have different deflections, in order to minimize drag, for a given constant control moment e.g. in cruise. Conversely, e.g. in a low-speed engine-out condition, with a given drag, it specifies the maximum control moment available.

Among the multitude of optimization methods available [1,2], the method of Lagrange multipliers was chosen; one of its major applications is in the calculus of variations [3], and hence in the analytical dynamics [4] of mechanical systems with multiple degrees-of-freedom subject to various types of constraints. The type of constraint in the present problem corresponds to anholonomic scleronomic in classical mechanics [5]. The method of Lagrange multipliers has several attractive features in the present application: (i) the magnitude of the final multiplier indicates how severely the constraint of pitch trim or constant lift affects cruise drag; (ii) the differences between the Lagrange multipliers of distinct control surfaces at each iteration indicates how far that particular state (or choice of control surface deflections) is from the final optimal state. There are other methods of optimization applicable to the selection of multiple control surfaces [6,7].

The comparison of the eight pitch trim strategies was initially motivated by (i) reduction of cruise drag, but raises other relevant and related issues, e.g.: (ii) if unchanged lift is required, to keep the same airspeed and altitude as for the untrimmed flight condition, can this be achieved with the same untrimmed angle-of-attack or is a higher value needed?; (iii) the largest control surface deflections required are sufficiently small to avoid concerns about adverse aeroelastic [8] or aerodynamic [9-13] effects, e.g. loss of control effectiveness [14,15] or control inversion, and shock wave formation and boundary layer separation?; (iv) the angle-of-attack of the aircraft and deflections of all control surfaces are within the range where linear or weakly non-linear methods can be applied with confidence? It should be borne in mind that there are many approaches to drag reduction [16,17], and the optimization of control surface deflections is just one of them.

2 TWO RECIPROCAL OPTIMIZATION PROBLEMS

Two reciprocal optimization problems in flight dynamics are: (i) for a given pitching moment, find the control surface deflections which minimize drag; (ii) for a given drag, find the control surface deflections which give maximum pitching moment. These two reciprocal problems correspond to: (i) the minimum drag due to pitch trim in cruise and (ii) the maximum pitching moment in an engine-out low-speed condition.

2.1 Minimum cruise drag due to pitch trim

Denoting by q the dynamic pressure:

$$q \equiv \frac{1}{2}\rho V^2, \tag{1}$$

the pitching moment per unit dynamic pressure and mean aerodynamic chord $\bar{c}$ is given by:

$$M^* \equiv M / q\bar{c} = \sum_{i=1}^{N} S_i\, C_{M_i}\left(\delta_i\right), \tag{2a}$$

where: (i) the sum extends to all N control surfaces, $i = 1,\ldots,N$, e.g. $i = 1$ is the elevator, and $i = 2,\ldots,N$ other surfaces which can be used for pitch control; (ii), S_i is the area, C_{M_i} the pitch control coefficient and δ_i is the deflection of the surface i. The trim drag per unit dynamic pressure is:

$$D^* \equiv D / q = \sum_{i=1}^{N} S_i\, C_{D_i}\left(\delta_i\right), \tag{2b}$$

where C_{D_i} is the dimensionless drag coefficient. The aim is to choose δ_i with $i = 0,\ldots,N$, so that, for a given pitching moment (2a), the drag (2b) is minimum.

2.2 Maximum pitching moment in engine-out condition

The condition of minimum drag (2b) requires that D^* be stationary

$$\mathrm{d}D^* = 0: \qquad \sum_{i=1}^{N} S_i\, C'_{D_i}\, \mathrm{d}\delta_i = 0, \tag{3a}$$

where $C'_{D_i} \equiv \mathrm{d}C_{D_i} / \mathrm{d}\delta_i$ and it is assumed that C_{D_i} depends only on the deflection of the corresponding surface δ_i (this will be generalized to coupled control surfaces in §4.1). However, the deflections δ_i are not independent, because they must keep the pitching moment (2a) constant:

$$M^* = const: \quad \sum_{i=1}^{N} S_i \, C'_{M_i} \, \mathrm{d}\delta_i = 0 . \tag{3b}$$

The reciprocal problem, e.g. corresponding to an engine-out condition, is for a given drag D^* to maximize the pitching moment:

$$\left(D^*_{\min}, M^* = const\right) \Leftrightarrow \left(D^* = const, M^*_{\max}\right), \tag{4}$$

and it leads to the set of same equations (3a,b). Thus both problems have a similar solution, i.e. lead to a set of optimal control surface deflections, which are determined next by the method of Lagrange multipliers.

2.3 Optimal control surface deflection and Lagrange multiplier

Introducing the Lagrange multiplier λ, the equation (3a) is multiplied by λ and (3b) is added:

$$\sum_{i=1}^{N} S_i \left(C'_{D_i} \lambda + C'_{M_i} \right) \mathrm{d}\delta_i = 0 . \tag{5a}$$

Now there are $N+1$ unknowns: (i) the N deflections δ_i of the control surfaces; (ii) the multiplier λ. If the deflections are taken as independent in (5a) then follow N optimization equations:

$$-\lambda \, C'_{D_i}\left(\bar{\delta}_i\right) = C'_{M_i}\left(\bar{\delta}_i\right), \tag{5b}$$

where the δ_i appears implicitly in the derivatives of the drag C'_{D_i} and moment C'_{M_i} coefficients; the subsidiary condition is the conservation of the pitching moment (3b), and it is the $(N+1)-th$ equation. From the $N+1$ equations (5b)+(3b), can be determined the N optimal deflections $\bar{\delta}_i$ and the Lagrange multiplier $\bar{\lambda}$. Once these are known, the minimum trim drag follows from:

$$D^*_{\min} = \sum_{i=1}^{N} S_i \, C_{D_i}\left(\bar{\delta}_i\right), \tag{6}$$

using the optimal deflections $\bar{\delta}_i$. The optimal Lagrange multiplier $\bar{\lambda}$ indicates how much the constraint of constant pitching moment M^* penalizes the minimum drag D^*, i.e. a large $\bar{\lambda}$ indicates a large effect on drag $D^*_{\min}$ to achieve the required pitching moment M^* and a small $\bar{\lambda}$ indicates a small effect on drag.

3 THE METHOD OF SOLUTION FOR THE OPTIMAL DEFLECTIONS

The direct (or minimum drag) and reciprocal (or maximum pitching) moment problems, lead to distinct optimal deflections (§3.1) because the subsidiary condition is different. The optimization conditions are the same (§3.2) and lend themselves to a similar iterative method of solution (§3.3).

3.1 Direct and reciprocal optimization problems

In the case of the reciprocal problem the same *N+1* unknowns, the *N* optimal control surface deflections $\bar{\bar{\delta}}_i$ and Lagrange multiplier $\bar{\bar{\lambda}}$ are determined from *N+1* conditions: (i) the same *N* optimization conditions (5b); (ii) the condition (3a) of constant drag. The maximum pitching control moment is given (2a) by:

$$M^*_{\max} = \sum_{i=1}^{N} S_i \, C_{M_i}\left(\bar{\bar{\delta}}_i\right). \tag{7}$$

Thus the direct (§2.1) [inverse (§2.2)] problems use the same *N* optimization conditions (5b), and a distinct subsidiary condition of constant pitching moment (3b) [constant drag (3a)] and thus lead to a different set of optimal deflections $\bar{\delta}_i\left(\bar{\bar{\delta}}_i\right)$ and a distinct Lagrange multiplier $\bar{\lambda}\left(\bar{\bar{\lambda}}\right)$:

$$\left\{(5b)+(3b)\,,\,\bar{\delta}_i\,,\,\bar{\lambda}\right\} \Leftrightarrow \left\{(5b)+(3a)\,,\,\bar{\bar{\delta}}_i\,,\,\bar{\bar{\lambda}}\right\} \tag{8}$$

In what follows the direct problem will be considered explicitly; similar reasonings would apply to the reciprocal problem. Note that the optimization conditions (5b) involve the first derivative of the drag C_{D_i} and pitch control C_{M_i} coefficients with regard to the deflections:

$$C'_{D_i} \equiv \mathrm{d}C_{D_i} / \mathrm{d}\delta_i\,, \qquad C'_{M_i} \equiv \mathrm{d}C_{M_i} / \mathrm{d}\delta_i\,, \tag{9a,b}$$

where it is assumed that the control surfaces are independent, i.e. each C_{D_i}, C_{M_i} depends only in the corresponding δ_i, and it is not affected by other δ_j with $i \neq j$. This restriction will be lifted in §4.1.

3.2 Iterative method of solution

The title of the paper and information about the author(s) should be formatted in the same way as the title of this guideline. The key to the solution of the direct and inverse optimization problems is to find the optimal deflections $\bar{\delta}_i$ and multiplier $\bar{\lambda}$ which satisfy the N optimization equations (5b) plus the subsidiary condition of constant pitching moment (3a). This is an implicit system, which can be solved iteratively (*Figure 1)* as follows: (i) start with equal deflections for all surfaces $\delta_i^{(0)} = \delta_0$, for the given pitching moment (2a):

$$\delta_i^{(0)} = \delta_0: \qquad M^* = \sum_{i=1}^{N} S_i \, C_{M_i}\left(\delta_0\right); \tag{10a}$$

(ii) the $N+1$ control equations (5b) specify $N+1$ first estimates of the multiplier:

$$\lambda_i^{(0)} = -C'_{M_i}\left(\delta_0\right) / C'_D\left(\delta_0\right), \tag{10b}$$

(iii) if all $\lambda_i^{(0)} \equiv \lambda^{(0)}$ are equal, we have the optimal solution $\delta_{0i} = \delta_0$, $\lambda = \lambda^{(0)}$, which satisfies all $N+1$ equations (2a,5b), implying that the minimum drag would occur for equal deflections of all control surfaces; (iv) most likely this is <u>not</u> the case, so if the $\lambda_i^{(0)}$ are not all equal, their arithmetic mean is taken as the next iteration:

$$\lambda^{(1)} = \frac{1}{N} \sum_{i=1}^{N} \lambda_i^{(0)}\,; \tag{11}$$

(v) substituting $\lambda^{(1)}$ in the N optimization equations (5b), gives the next iteration for the deflections $\delta_i^{(1)}$, which are generally distinct; (vi) substituting the $\delta_i^{(1)}$ in the pitching moment (2a) leads to a value $M^{(1)}$, which is the optimum if $M^{(1)} = M_*$ has the required value M_*; (vii) if not $\lambda^{(1)}$ is modified to $\lambda^{(2)}$ and the process continued until $M^{(n)} \to M_*$. The modification process for the Lagrange multiplier at the iteration $n+1$ could be as in (11) the arithmetic mean of the preceding n^{th} iteration.

3.3 Linear and non-linear optimization equations

The critical step in iterative method of solution is to solve the optimization equation (5b) for the deflections δ_i. This is analyzed next. If the drag C_{D_i} and control C_{M_i} coefficients are linear in the deflections δ_i, then (9a,b) are constants, and generally there is not a single value λ of the Lagrange multiplier which will satisfy all *N* relations (5b). Thus there is no optimum. It follows that optimal deflections exist only if the drag or control coefficients are non-linear functions of the control surface deflections, e.g.:

$$C_{D_i}\left(\delta_i\right) = C_{D_{0i}} + C'_{D_{0i}}\,\delta_i + \frac{1}{2}\left(\delta_i\right)^2 C''_{D_{0i}}\,, \tag{12a}$$

$$C_{M_i}\left(\delta_i\right) = C_{M_{0i}} + C'_{M_{0i}}\,\delta_i + \frac{1}{2}\left(\delta_i\right)^2 C''_{M_{0i}}\,, \tag{12b}$$

in a quadratic case. In this case the optimization conditions are specified substituting (12a,b) in (5b):

$$-\lambda\left(C'_{D_{0i}} + \delta_i\, C''_{D_{0i}}\right) = C'_{M_{0i}} + \delta_i\, C''_{M_{0i}}\,, \tag{13a}$$

and can be solved for the deflections:

$$-\delta_i = \frac{C'_{M_{0i}} + \lambda C'_{D_{0i}}}{C''_{M_{0i}} + \lambda C''_{D_{0i}}}\,; \tag{13b}$$

thus the optimum exists, as specified by (13b).

4 EXTENSION TO COUPLED SURFACES AND ADDITIONAL CONSTRAINTS

The preceding method is extended in two ways: (i) to coupled control surfaces (§4.1), such that the drag C_{D_i} and control C_{M_i} coefficients of surface *i* depend on the defections δ_j of other surfaces; (ii) to additional constraints (§4.2), e.g., a condition of constant lift. The two generalizations can be taken together, and it is important to note that the optimization problem depends on the position of c.g., allowing an approximation for (§4.3) small deviation.

4.1 Optimization condition for the coupled control surfaces

In the case of coupled control surfaces the drag coefficient C_{D_i} of surface *i* depend on the defections δ_j of all control surfaces:

$$D_* = \sum_{i=1}^{N} C_{D_i}\left(\delta_j\right) S_i \,, \tag{14a}$$

and the condition of minimum (or constant) drag:

$$0 = \mathrm{d}D_* = \sum_{i,j=1}^{N} C'_{D_{ij}} S_i \,\mathrm{d}\delta_j \,, \tag{14b}$$

involves a matrix of first-order derivatives (15a):

$$C'_{D_{ij}} \equiv \partial C_{D_i} / \partial \delta_j \,, \qquad C'_{M_{ij}} \equiv \partial C_{M_i} / \partial \delta_j \tag{15a,b}$$

and likewise for the pitching moment (15b). If follows that the optimization conditions (5b) are now:

$$0 = \mathrm{d}M_* + \lambda \,\mathrm{d}D_* = \sum_{i=0}^{N} S_i \sum_{j=1}^{N} \left(C'_{M_{ij}} + \lambda C'_{D_{ij}} \right) \mathrm{d}\delta_j \,; \tag{16a}$$

since the variations of the deflections cannot be all zero, the determinant must vanish:

$$\left(\mathrm{d}\delta_1, \mathrm{d}\delta_2, ..., \mathrm{d}\delta_N\right) \neq \left(0,0,...,0\right) : \det\left(C'_{M_{ij}} + \lambda C'_{D_{ij}} \right) = 0 \,. \tag{16b}$$

Thus minus the Lagrange multiplier $-\lambda$ is an eigenvalue of the pair of matrices $C'_{M_{ij}}$ and $C'_{D_{ij}}$. If the eigenvalues are distinct then the eigenvectors are orthogonal. In this orthogonal frame, with indices *a*,*b*, the matrices became diagonal:

$$C'_{M_{ab}} = C'_{M_a} I_{ab} \,, \qquad C'_{D_{ab}} = C'_{D_a} I_{ab} \,, \tag{17a,b}$$

where I_{ab} is the identity matrix; thus (16a) show that:

$$0 = dM_* + \lambda \, dD_* = \sum_{a=1}^{N} S_a \, d\delta_a \left(C'_{M_a} + \lambda \, C'_{D_a} \right); \tag{18}$$

the controls are now decoupled as in (5b). Hence the analysis of (§3.3) applies again, to show that an optimum exists for non-linear controls, e.g. (13b) in the quadratic case (12a,b).

4.2 Flight at varying speed/altitude or constant lift

The lift per unit dynamic pressure induced by the control surfaces is given by:

$$L* = L / q = \sum_{i=1}^{N} C_{L_i} S_i \,, \tag{19}$$

where C_{L_i} is the lift coefficient of control surface *i*. In the preceding optimization problem, the optimal deflections $\bar{\delta}_i$ would generally lead to a change in lift to $L*$, implying flight at a different dynamic pressure (1), i.e. either speed or altitude or both will change, viz.: (i) if lift was increased $\bar{L}* > L*$, then the aircraft would descend to a lower altitude, corresponding to a larger atmospheric mass density ρ, or fly faster, leading to a combination such that lift equals weight again; (ii) if lift was decreased, then the aircraft would fly at higher altitude, or slower.

If it is required to keep flight altitude and speed, then the constraint of constant lift can be introduced via an additional Lagrange multiplier μ relative to (16a), viz.:

$$0 = \mathrm{d}M_* + \lambda \,\mathrm{d}D_* + \mu \,\mathrm{d}L_* \,, \tag{20a}$$

and the optimization condition (5b) would be replaced by:

$$0 = C'_{M_i} + \lambda \, C'_{D_i} + \mu \, C'_{L_i} \,, \tag{20b}$$

for decoupled control surfaces (§3.3). In the case of coupled control surfaces (§4.1) the relations (20b) would hold after diagonalization, similar to (17a,b) also for $C'_{L_{ab}}$. The iterative method of solution (§3.2) would again apply, with two multipliers λ, μ. Any additional constraint, would add another multiplier.

4.3 Effec on the optimization of large and small c.g. shifts

The preceding calculations involve the c.g. position in two ways: (i) the moment arm L_i can be calculated for l_i a given c.g. position (e.g. 25% of mean aerodynamic chord) and then corrected for the c.g. deviation Δx_{cg} from this position (27a):

$$L_i = l_i - \Delta x_{cg} \quad ; \qquad \left(\Delta x_{cg}\right)^2 << \left(l_i\right)^2 ; \tag{21a,b}$$

(ii) if the c.g. position shift is small (21b) relative to the moment arm, then the aerodynamic coefficients are not affected. The pitching moment (2a) is now taken for unit dynamic pressure:

$$M^{**}/q = \sum_{i=1}^{N} L_i \, S_i \, \bar{C}_{Mi}\left(\delta_i\right), \tag{22}$$

and the moment arms L_i introduced instead of the mean aerodynamic chord $\bar{c}$; note that this changes the value of the pitching moment coefficient to $\bar{C}_{Mi}$. Using (13b) with C_{M0i} replaced by $L_i \, \bar{C}_{M0i}$, leads to:

$$-\delta_i = \frac{l_i \bar{C}'_{M_{0i}} + \lambda C'_{D_{0i}} - x_{cg} \bar{C}'_{M_{0i}}}{l_i \bar{C}''_{M_{0i}} + \lambda \bar{C}''_{D_{0i}} - x_{cg} \bar{C}''_{M_{0i}}}, \tag{23}$$

where the aerodynamic coefficient may depend on the c.g. position. For small c.g. deviations (21b), the optimal deflections (23) linearize:

$$-\delta_i = \frac{l_i \bar{C}'_{M_{0i}} + \lambda C'_{D_{0i}}}{l_i \bar{C}''_{M_{0i}} + \lambda C''_{D_{0i}}} \times \\ \times \left\{ 1 + x_{cg} \left[\frac{\bar{C}''_{M_{0i}}}{l_i \bar{C}''_{M_{0i}} + \lambda C''_{D_{0i}}} - \frac{\bar{C}'_{M_{0i}}}{l_i \bar{C}''_{M_{0i}} + \lambda C'_{D_{0i}}} \right] \right\} \tag{24}$$

where the aerodynamic coefficients are calculated for the reference c.g. position $x_{cg} = 0$.

5 RESULTS

The optimization procedure is iterative, and needs an initial condition to start. In order to give a choice of starting values for the optimization procedure, three initial conditions are considered: equal deflection and all control surfaces, which gives moderate deflections, but uses some control surfaces less effective than the elevator; deflecting the elevator alone, uses only the most effective control surface, but can lead to a large deflection; limiting the maximum deflection, implies using, in addition to the elevator, some more, possibly not all, control surfaces. It may be advisable to limit control surface deflections on cruise for at least two reasons: (i) a large deflection of an inboard control surface, say a centerbody elevator, could lead to shock formation and boundary layer separation; (ii) a large deflection of an outboard control surface, say an aileron, could cause aeroelastic effects, like loss of control effectiveness or even control inversion.

The minimization of cruise drag with pitch trim and unchanged lift is considered for a flying-wing configuration, using eight strategies. Of the four non-optimal strategies, only strategy II of using the centerbody elevator alone leads to drag reduction, albeit with a large deflection and increased angle-of-attack.

There is drag increase for the strategies: (I) of equal deflection of all control surfaces; (III) preferential deflection of inner control surfaces with an aeroelastic limit of 7.5º; (IV) equal contribution to lift for all control surfaces. The two optimal strategies V using Lagrange multipliers after a few interactions give: (VA) a drag reduction for a good initial condition like strategy II; (VB) a drag increase for a poor initial condition like strategy I. A sub-optimal strategy VI of using multiples of optimal deflections, with a multiplication factor determined by lift equilibrium, requires a larger angle-of-attack, and thereby increases drag. The strategy VII of deflecting all control surfaces in two groups to minimize trim drag, leads to a larger angle-of-attack, and thereby increases drag. The strategy VIII of using the two most effective control surfaces with opposite drag slopes gives the best compromise: (i) lift balance with unchanged angle-of-attack; (ii) second best drag reduction with small control deflections. The eight strategies I to VII, and twelve sub-strategies (A and B for VII and A,B,a,b for V) are summarized in *List I*. Since the deflection of control surfaces upwards (51b) decreases lift, the angle-of-attack is increased (51a), and hence also the drag, as shown on *Table VIII.* In the case of the strategy I of equal deflection of all control surfaces, the condition of cruise lift equilibration increases the trim drag penalty from 89 to 197 drag counts. The opposite will be shown next for the strategy II of elevator deflection only, when the cruise lift equilibration condition improves the pitch trim drag benefit from 16 to 18 counts, as shown next.

It is seen from *Table 2* that the sub-optimal strategy VI, of non-optimal use of optimal deflections from strategy VB, improves over strategy VB, but not over the better initial condition in strategy VA. This result is not surprising, since it has been shown that pitch trim is compatible with cruise drag reduction. The optimal strategy V may not be the best in practice, if parameter limitations prevent from reaching the optimum; in the present case the deflections are limited to small values and the aerodynamic coefficients, and their first-and second-order derivatives, have a limited range of variation. The best results obtained so far were for a simpler non-optimal strategy of elevator deflection only. This result could be due to the linearized aerodynamic data used. In general, a simple non-optimal strategy can give good results if it incorporates the lessons learned from optimal strategies, as suggested by strategy VI, which is further improved next.

The *Table 3* ranks the strategies according to the most desirable features, viz.: (i) lowest cruise drag; (ii) smallest angle-of-attack; (iii) smallest control deflections in the sense of smallest average of modulus of largest and smallest deflection $\bar{\delta} \equiv \left(|\delta_{\min}| + |\delta_{\max}|\right)/2$; (iv) linear aerodynamics works best for smallest angle-of-attack α and smallest mean deflection $\bar{\delta}$; (v) simplicity of the computational algorithm, viz. least number of variables and least iterations. The strategy VIII has been shown to have five relative advantages: (i) it has the second lowest trimmed cruise drag of all eight strategies and twelve sub-strategies considered: (ii) the trimmed angle-of-attack is unchanged from the untrimmed value, and hence is the lowest and small; (iii) the pitching moment is thus small, and leads to moderate (5th smallest) control surface deflections; (iv) the small angle-of-attack and control surface deflections imply that linear aerodynamics can give reliable results (3rd best); (V) the strategy is easy to apply (3rd best). The other pitch trim strategies considered in *Table 3* do not have one or more of these attributes. The deflections of five sets of central surfaces, and their effects on angle-of-attack, pitching moment, lift and drag are shown in Tables 1 and 2 for the eight distinct strategies. The comparison of the 8 pitch trim strategies in Table 3 shows that the most complex or demanding is not necessary the best; it is possible to find a relatively simple strategy VIII, which achieves pitch trim drag reduction relative to the untrimmed flight condition, while also keeping the low untrimmed angle-of-attack and using moderate deflections of only the two most effective control surfaces.

6 THE METHOD OF SOLUTION FOR THE OPTIMAL DEFLECTIONS… DISCUSSION

In a conventional aircraft configuration the elevator or all-flying tail is the only effective pitch control surface. The elevator must be large enough to provide adequate pitch control authority at low speed, when large deflections are possible. At high speed, when aerodynamic and/or aeroelastic effects may limit deflections, the higher dynamic pressure may compensate, and not be a "sizing" criterion requiring large area, with aerodynamic, aeroelastic, weight or actuator problems. In any case the compromise in elevator sizing is straightforward, once the required pitching moment and possible lever arm are known. The BWB or flywing wing configuration has the potential to provide pitch trim with several combinations of control surface deflections. There are simple choices with obvious consequences but it is not clear what is the best compromise, e.g.: (i) the strategy II of deflecting the centerbody elevator only can lead to a large deflection; (ii) using all control surfaces with the same deflection (strategy I) or same lift (strategy IV) does not exploit the most effective controls; (iii) deflecting preferentially the inner control surfaces with an aeroelastic limit (strategy III) may not be the best compromise among the preceding choices.

The optimization methods may or may not do better than the simples non-optimal strategies depending on whether the drag has a single global minimum or several local minima. If there are several local minima convergence will occur to that closest to the initial condition. The influence of the initial condition on the result of the optimization may not be obvious. It may be difficult to know if a local minimum is lower that neighbouring ones or is the global minimum. Also the optimization algorithm may require data, like second-order control derivatives which is not accurate, leading to a possible dilemma: (i) too few iterations do not converge closely enough to the optimum; (ii) too many iterations accumulate too large an error around the optimum. It may also occur that the local minimum is outside the acceptable range of control deflections, and the optimum is not a local minimum but rather the best value on a boundary. Thus optimal methods have strengths and weakness which may be different from simple methods and a combination of the two may be more effective than either of them.

A careful examination of control derivatives is essential to understand how an optimization procedure is going to work, or what are the consequences of a simple strategy. In the case of pitch trim for cruise the starting ideas seems to be to select the most effective control surfaces, in the sense of combining the largest possible pitching moment and drag reduction with small deflections; it is also important to achieve lift balance, so that it is not necessary to increase the angle-of-attack for trimming purposes. A larger angle-of-attack increases drag, may require a larger pitching moment, and thus works against having selected trim with the "most effective" control surfaces; thus the most effective control surfaces should be chosen to (i) provide pitching moment, (ii) reduce drag and (iii) balance lift. In the present example it was possible to achieve pitch trim while reducing drag relative to an untrimmed flight condition, also not changing angle-of-attack; the latter contributes to smaller control deflections, avoiding undesirable aeroelastic effects, and staying within the limits of linear or weakly non-linear aerodynamics and control methods.

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List 1 – Pitch trim strategies
I – Collective: Deflect all trailing edge control surfaces by the same amount:
non-optimal, cruise drag increase.
II – Elevator: Deflection of elevator alone: non-optimal, best cruise drag reduction, but large deflection.
III – Inboard: Preferential deflection of inboard control surfaces, up to a given limit: non-optimal, cruise drag increase for 7.5º limit.
IV – Equidistribution: Equal contribution to lift from each control surface:
non-optimal, cruise drag increase.
V – Optimal: Deflection of each control surface to minimize cruise drag: does not achieve optimum for small number of iterations, when it depends on initial condition:
IVA: Drag reduction starting with strategy II;
IVB: Drag increase starting with strategy I .
- Compatible with cruise equilibrium by two alternative approaches:
IVa: Extension to two Lagrange multipliers;
IVb: Iteration using angle-of-attack equal to mean values of angles-of-attack for (i) constant lift and (ii) zero pitching moment.
VI – Scaling: Optimal deflections multiplied by scale factor to achieve cruise equilibrium: sub-optimal, cruise drag increase and increased angle-of-attack.
VII – Split: Split the control surfaces into two groups to minimize drag: (i) those with positive drag slope deflect downward; (ii) those with negative drag slope deflect upward. The deflections of each group are determined by lift equilibrium and pitch trim: increases angle-of-attack and also cruise drag.
VIII – Selective: Select the best control surface from each group (i) and (ii) in strategy VII, using as criteria largest (a) modulus of slope of drag coefficient, (b) slope of pitching moment coefficient or (c) product of the two: second best cruise drag reduction at low unchanged angle-of-attack, with small deflections, for linear aerodynamics with minimum risk of aeroelastic effects: the best strategy.

Figure 1

Block diagram of iterative method of solution of optimization problem using

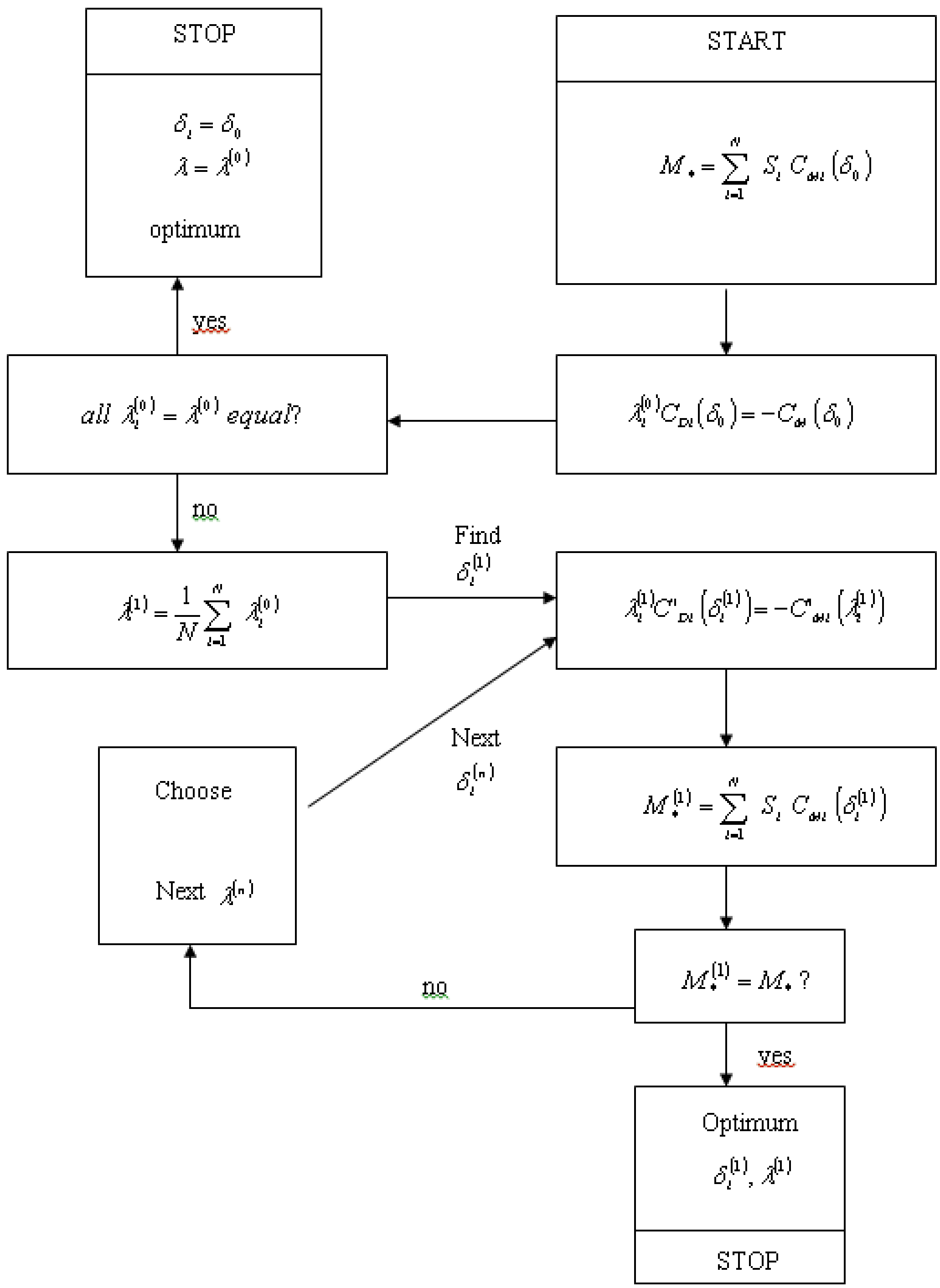

Table 1 - Effect of lift equilibrium on pitch trim strategies

Strategy	I		II		IV
Concept	Equal deflection of all control surfaces		Deflection of elevator only		Equal contribution to lift from each control surface
Cruise lift equilibrium	No	Yes	No	Yes	Yes
C_L	0.10588	0.10588	0.10588	0.10588	0.10588
α	0.71789º	1.8406º	0.71789º	2.0727º	2.27940º
C_M (δ=0º)	0.02441	0.04971	0.02441	0.05601	0.05968
δ_1	4.46780º	1.7200º	8.7977º	10.367º	6.6990º
δ_2	4.46780º	1.7200º	0.0º	0.0º	7.6277º
δ_3	4.46780º	1.7200º	0.0º	0.0º	2.7478º
δ_4	4.46780º	1.7200º	0.0º	0.0º	14.971º
δ_5	4.46780º	1.7200º	0.0º	0.0º	5.3920º
C_D	0.00659	0.00766	0.00553	0.00551	0.01089
ΔC_D	+89	+197	-16	-18	+520
$\Delta C_D/C_D$ (%)	+15.6	+34.5	-2.8	-3.2	+91.4

Table 2 - Optimal, sub-optimal and non-optimal pitch trim strategies

Strategy	VA		VB		VI	VII		VIII
Type	Optimal		Optimal		Sub-Optimal	Non-Optimal		Non-Optimal
Concept	Optimal deflections to minimize drag; initial elevator deflection only		Idem; initial equal deflections		Multiples of optimal deflections	Direction of deflection opposite to drag slope		Selection of most effective control surfaces
Constant lift	No		No		Yes	Yes		Yes
Deflections	Initial	Final	Initial	Final		VIIA	VIIB	
δ_1	8.9798º	7.4047º	4.4080º	4.125º	17.912º	-2.8825º	2.8588º	3.7402º
δ_1	0.0º	7.5882º	4.4080º	0.02º	0.0868º	-2.8825º	2.8588º	0.0º
δ_1	0.0º	-0.6048º	4.4080º	1.68º	8.1637º	-2.8825º	2.8588º	0.0º
δ_1	0.0º	-1.4860º	4.4080º	0.01º	0.0434º	-2.8825º	2.8588º	0.0º
δ_1	0.0º	-7.2529º	4.4080º	-3.685º	-16.002º	24.77º	0.0º	-8.6295º
α	0.71789º		0.71789º		3.1267º	0.71789º	2.1313º	0.71789º
C_D	0.00561		0.00770		0.00635	0.00651	0.00787	0.00554
ΔC_D (counts)	-8		+201		+66	+82	+218	-15
$\Delta C_D/C_D$ (%)	-1.4		+35.3		+11.6	+14.4	+38.3	-2.6

Table 3- Ranking of Desirable features of pitch trim strategies. Lagrange multipliers.

Strategy	Concept	Cruise drag counts (Lowest)	Angle-of-attack (Lowest)	Central surface deflections (lowest average of module)	Linear aero-dynamics (Lowest α and lowest δ)	Simplicity
I	Equal deflections	766 (+97) (7th)	1.8406° (2nd)	1.7200° (2nd)	6th	1st
II	Deflection of elevator only	551 (-18) (1st)	2.0727° (3rd)	10.367° (7th)	7th	2nd
III	Preferential inboard surfaces with 7.5° limit	659 (+100) (untrimmed) (6th)	0.71789° (1st)	8.9798° (6th)	4th	7th
IV	Equal lift contributions from each surface	1089 (+420) (9th)	2.2794° (5th)	2.7478° to 14.971° (7th)	9th	6th
V Optimal Deflection	A: good start	561 (-8) (untrimmed) (3rd)	0.71789° (1st)	-7.2529° to +7.4047° (5th)	3rd	9th
	B: bad start	635 (+66) (untrimmed) (4th)	0.71789° (1st)	-3.685° to + 4.125° (3rd)	1st	10th
VI	Multiples of optimal deflection	635 (+66) (4th)	3.1267° (6th)	-16.002° to + 17.912° (9th)	10th	8th
VII	Direction of deflection opposite to drag slope	651 (+82) (5th)	0.71789° (1st)	-2.8825° to + 24.727° (10th)	5th	4th
		787 (+218) (8th)	2.1313° (4th)	0.0° to + 2.8588° (1st)	8th	5th
VIII	Selection of most effective control surfaces (1st)	554 (-15) (2nd)	0.71789° (1st)	-8.6295° to + 3.7402° (4th)	2nd	3rd