The constructal principle and characterial law, hyperbolic, of design, a new paradigm of aerospace systems

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ABSTRACT

The paper set the physical and mathematical foundations of the global, holistic and natural model of performance of an aerospace propulsion system from turbojet engine family, having as foundation the law of hyperbolic harmony of the universe, taking as a basis, the duality between the thrust, via the gas dynamics function of momentum and mass, by gas dynamics flow function. We get, thus, another point of view on thrust but, also a complete solution of David Hilbert's fourth problem. It is also found

- introduction of the principle of hyperbolic harmony by combining universal principles well known of complementarity and harmony;
- introduction and use of specific concepts for turbojet engine;
- using, instead of absolute amounts of intake, mass, thermal and geometric coefficients, in evaluating specific thrust force;
- development of new solutions, modern of non pollutant turbojet engines.

The fundamental idea of this paper is to calculate the specific thrust force of a propulsion system (thrust) class of air-jets engine, with application to turbojet simple flow, based on the definition and use of harmonic complementary patterns between gas dynamics functions of impulse and thrust.

1 INTRODUCTION

In recent years, it is often talking more about design, a new concept, which is more comprehensive then drawing or projection. Design is a transdisciplinary concept comprising triad, projection, aesthetics and architecture.
Design is a different way to see a thing.
If, until now, the emphasis was on analyzing the components of a system, on integration, now is highlighted what is essential, in a system, meaning, what it represent and characterize, namely what makes it natural, simple, beautiful and economically.
This global perspective involves the establishment of constructal principles and characterial law.
Undoubtedly, in a material world, dual, constructal principle can only have the same nature, dual.
Regarding the characterial law in a natural world, harmonic and characterial, the law can only be the expression of complementarity, namely hyperbolic.
If they add gold proportions, ie aesthetics, we can speak about simplicity from other fundamental aspect of design.
Therefore simplicity, naturalness and harmony define design.

2 MATHEMATICAL BASES OF HOLYSTIC MODEL

Further, will consider a schematic diagram [1] of a generalized nozzle, as in Figure. 1

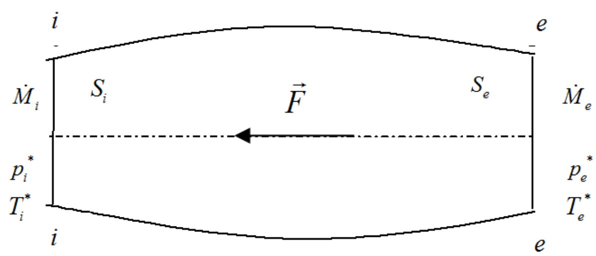

Figure 1: Schematic diagram of a generalized nozzle

Where the characteristic parameters nature such as
- mass flow, $\dot{M}$;
- p^*, T^*, total pressure and temperature;
- geometric area , S ,

define the two states of the fluid, corresponding to the
- entry to the nozzle, index *i*;
- exhaust from the nozzle, index *e*.

Of course, the force developed by the nozzle, forward $\vec{F}$, consists of
- thrust elements, $\vec{T}_j$;
- propulsion elements, $\vec{P}_k$;
- compound elements, thrust-propulsion, $\vec{C}_l$.

As a result, the summed force [2] becomes

$$\vec{F} = \sum_{j=1}^{n} \vec{T}_j + \sum_{k=1}^{m} \vec{P}_k + \sum_{l=1}^{p} \vec{C}_l \qquad (1)$$

In general [3], the amount of force developed by the nozzle, F_i , can be expressed by the formula [5]

$$F_i = F_{ce} - F_{ci}, \qquad (2)$$

F_c symbolizes the current local force, where the fluid jet meets gas dynamics opposition, like external atmospheric pressure p_H .

By definitions, the current local force can be written as

$$F_c = F_{cv} - p_H \cdot S\,, \qquad (3)$$

where F_{cv} represents the local force of the current in vacuum, in the absence of atmospheric.
It is known that the local force of the current in vacuum, is the sum of two components, static, $p \cdot S$ and dynamic, $\dot{M} \cdot V$, that is

$$F_{cv} = \dot{M} \cdot V + p \cdot S, \tag{4}$$

where V and p are the absolute local speed, respectively, local static pressure of the fluid.
It is known the expression of the local force of the current in vacuum, based on gas dynamics function of impulse, $z(\lambda)$,

$$F_{cv} = c_f \cdot \dot{M}\sqrt{T^*} \cdot z(\lambda), \tag{5}$$

the constant function c_f, is

$$c_f = \sqrt{2 \cdot \frac{k+1}{k} \cdot R}, \tag{6}$$

where k is adiabatic exponent of fluid evolution, R is gas constant and λ is Chaplygin number, counterpart of Mach number, relative to flow critical conditions, minimum.
It takes into account that gas dynamics function of impulse [2] is expressed by

$$z(\lambda) = \frac{1}{2}\left(\lambda + \frac{1}{\lambda}\right) \tag{7}$$

and can be plotted based on λ, as in Figure 2.

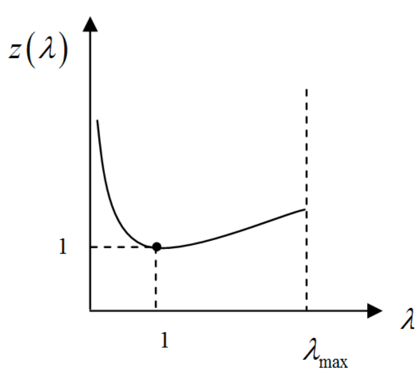

Figure 2: Gas dynamics function of impulse

Given the important role played by fluid mass flow in achieving thrust force [3], we use the known local relationship

$$\dot{M} = c_m \cdot \frac{p^*}{\sqrt{T^*}} \cdot S \cdot q(\lambda), \tag{8}$$

where
- c_m is mass flow constant,

$$c_m = \sqrt{\frac{R}{k}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}};$$

- $q(\lambda)$ the gas dynamics function [4] of the mass flow,

$$q(\lambda) = \lambda\left(\frac{k+1}{2} - \frac{k-1}{2} \cdot \lambda^2\right)^{\frac{1}{k-1}}. \tag{9}$$

The graphic, based on λ, the gas dynamics function of the flow has the aspect from Figure 3.

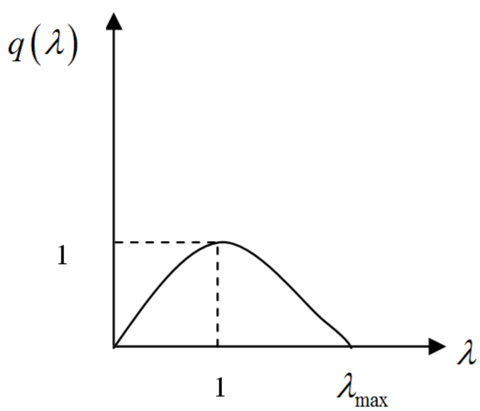

Figure 3: Gas dynamics function of the flow

Applying equation (5), in the fundamental sections of the generalized nozzle, *e-e* and *i-i*, and the conditions given by the relation (3), then the current local forces becomes primary forms [5].

$$F_{ci} = c_{fi} \cdot \dot{M}_i \cdot \sqrt{T_i^*} \cdot z(\lambda_i) - p_H \cdot S_i \tag{10}$$

and

$$F_{ce} = c_{fe} \cdot \dot{M}_e \cdot \sqrt{T_e^*} \cdot z(\lambda_e) - p_H \cdot S_e \,. \qquad (11)$$

So, based on relation (2), the developed force becomes

$$F_i = c_{fi} \cdot \dot{M}_i \left[\frac{c_{fe}}{c_{fi}} \cdot \frac{\dot{M}_e}{\dot{M}_i} \cdot \sqrt{\frac{T_e^*}{T_i^*}} \cdot \frac{z(\lambda_e)}{z(\lambda_i)} - 1 \right] - p_H \cdot S_i \left(\frac{S_e}{S_i} - 1 \right). \qquad (12)$$

To simplify writing, we define the input coefficient $\bar{X}$,

$$\bar{X} = \frac{X_e}{X_i},$$

where X is an arbitrary quantity.

As such, it is stated further

- $\bar{\dot{M}} = \dfrac{\dot{M}_e}{\dot{M}_i}$, coefficient of mass rate contribution;
- $\bar{T}^* = \dfrac{T_e^*}{T_i^*}$, coefficient of temperature contribution;
- $\bar{p}^* = \dfrac{p_e^*}{p_i^*}$, coefficient of mechanical (or pressure) contribution;
- $\bar{S} = \dfrac{S_e}{S_i}$, coefficient of geometric contribution;
- $\bar{c}_f = \dfrac{c_{fe}}{c_{fi}}$, coefficient of constant impulse;
- $\bar{c}_m = \dfrac{c_{me}}{c_{mi}}$, coefficient of constant mass.

With this notations, relation (12), can be written as

$$F_i = c_{fi} \cdot \dot{M}_i \left[\bar{c}_f \cdot \bar{\dot{M}} \cdot \sqrt{\bar{T}^*} \cdot \frac{z(\lambda_e)}{z(\lambda_i)} - 1 \right] - p_H \cdot S_i \left(\bar{S} - 1 \right). \qquad (13)$$

Regarding fluid mass flow, it can be played in two main sections by

$$\dot{M}_e = c_{me} \cdot \frac{p_e^*}{\sqrt{T_e^*}} \cdot S_e \cdot q(\lambda_e)$$

and

$$\dot{M}_i = c_{mi} \cdot \frac{p_i^*}{\sqrt{T_i^*}} \cdot S_i \cdot q(\lambda_i) \ .$$

Under these conditions, the coefficient of mass input has the form

$$\bar{\dot{M}} = \bar{c}_m \cdot \frac{\bar{p}^*}{\sqrt{\bar{T}^*}} \cdot \bar{S} \cdot \frac{q(\lambda_e)}{q(\lambda_i)} \ . \tag{14}$$

Obviously, from (13)

$$z(\lambda_e) = z(\lambda_i) \cdot \frac{1}{\bar{c}_f} \cdot \frac{1 + \dfrac{F_i + p_H S_i (\bar{S} - 1)}{\dot{M}_i \cdot c_{fi}}}{\bar{\dot{M}} \cdot \sqrt{\bar{T}^*}} \tag{15}$$

and, from (14),

$$q(\lambda_e) = q(\lambda_i) \cdot \frac{1}{\bar{c}_m} \cdot \frac{\bar{\dot{M}} \cdot \sqrt{\bar{T}^*}}{\bar{p}^* \cdot \bar{S}} \ . \tag{16}$$

It is, therefore, necessary to settle the conditions which represent relations between the two gas dynamics functions in order to express the idea of harmonic flow out of the generalized nozzle section.
The originality of this paper is to find simple ways of elimination, taking as a basis, the harmony [6] [7] that unites the two gas dynamic functions, $z(\lambda)$ and $q(\lambda)$, under the principle of complementarity, expressed by different laws.
Taking into account the impulse and debit functions, that define the shape and mass, in general, their dual complementarity leads us to three main harmonic laws [8]

- Linear law,

$$q(x) + z(\lambda) \approx 2; \tag{17}$$

- Parabolic law,

$$z(x) = c_1 q(\lambda) + \frac{c_2}{q(\lambda)} + c_3; \tag{18}$$

- Hyperbolic law,

$$q(x) \cdot z(\lambda) \cong 1. \tag{19}$$

Plotting these laws images obtained are those from Figure 4.

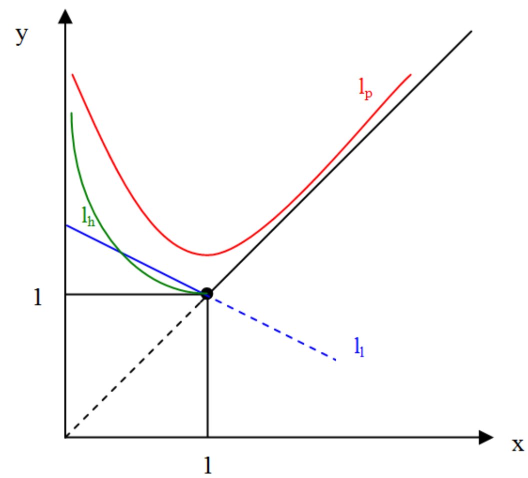

Figure 4: Harmonic law

3 CASE STUDY

Next, in order to determine the best characterial law, suitable to the dual system of a jet engine, we establish the relationships that define specific engine thrust, F_{SP_i}, for every law specified above.
Thus, for

- Linear law, F_{SP_1}, the expression form is

$$F_{SP_I} = \alpha_1\left(\dot{\bar{M}}\cdot\sqrt{\bar{T}^*}-1\right)+\beta_1\left(\frac{\dot{\bar{M}}^2\cdot\bar{T}^*}{\bar{p}^*\cdot\bar{S}}-1\right)+\gamma_1\left(\bar{S}-1\right)+\delta_1 . \tag{20}$$

- Parabolic law F_{SP_2}, the form

$$F_{SP_{II}} = \alpha_2\left(\dot{\bar{M}}\cdot\sqrt{\bar{T}^*}-1\right)+\beta_2\left(\frac{\dot{\bar{M}}^2\cdot\bar{T}^*}{\bar{p}^*\cdot\bar{S}}-1\right)+\gamma_2\left(\bar{S}-1\right)+\delta_2\left(\bar{p}^*\cdot\bar{S}-1\right), \tag{21}$$

- Hyperbolic characterial law F_{SP_3}, is

$$F_{III} = \frac{1}{c_{mi}}\cdot\frac{\dot{\bar{M}}\cdot\sqrt{\bar{T}^*}}{q\left(\lambda_i\right)}\left[c_{vi}\left(\bar{c}_v\cdot\bar{p}^*\cdot\bar{S}-1\right)-\frac{p_H}{p_i^*}\left(\bar{S}-1\right)\right]. \tag{22}$$

It is interesting that, assuming similar conditions at the entrance of the engine, identical values for the fundamentals parameters and coefficients of the system, the values obtained are

$F_{SP1} \approx 859$ m/s ;

$F_{SP2} \approx 945,75$ m/s ;

$F_{SP3} \approx 810$ m/s .

4 CONCLUSIONS

Based on these results, we mention some interesting conclusions. So

- All holistic models allowed us evaluation of the overall performance of the engine, taking into account, the relations between engine components;
- After the appearance of the three harmonic laws, closest to reality, which leads to closest results with the one existing in literature, is hyperbolic law;
- The best harmonization law is the hyperbolic law, proven by simplicity of specific force expression results, which confirmed, once again, that the test of truth is simplicity;
- For characterial hyperbolic law observe that

$$F_{SP} = f\left(\overline{p}^{*}, \overline{S}\right),$$

meaning fundamental performance of a nozzle or, furthermore, of a engine, does not depend on the mass input , which is necessary condition.

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