AIR TRAFFIC FLOW OPTIMISATION WITH TRAJECTORY UNCERTAINTY

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ABSTRACT

A four dimensional multi-criteria optimization model is proposed. The cooperative air traffic flow optimisation framework incorporates wind patterns and on-time departure and arrivals subject to allocated times. Information relevant to Air Traffic Management (ATM) stakeholders, such as aircraft performance, flight plans, weather, aircraft arrival schedules, etc. will be integrated into the optimisation model. It is intended to establish an ATM system that generates solutions based on intent and anticipation rather than short-term and local solutions. The model will seek a global optimum subject to re-optimisations at pre-set intervals. The re-optimisation will consider both old and new data in its decision making. The intention is to minimise the level of uncertainty associated with flights, specifically long haul flights.

1 INTRODUCTION

Sustainability has been a strong driving factor for advancements in aerospace technology. From improvements to aircraft design through to engine performance, the aviation industry has seen a dramatic change over the past few decades. As the fastest and cheapest means of long distance travel, passenger demand and freight has demonstrated an exponential growth with future projections anticipating a similar trend. Increased passenger demand encouraged the production of more aircraft and the construction and expansion of airports to increase available capacity. This coincided with the establishment of more airlines as air transport moved from a government owned industry to a privately owned airline industry in the late 1990’s [24].

The role of Air Traffic Flow Management (ATFM) is to monitor and manage traffic flow conditions and demand. Detailed analytical attention has been paid to the development of ATFM systems and procedures for the past few decades. Bielli’s 1982 study into network models supporting automated systems focuses on traffic congestion forecasts and congestion prevention forecasts [16]. Since then, the concern for Air Traffic Management (ATM) systems competently managing increased traffic demand has not receded. ATM can be described as the combination of ATFM, Air traffic Control (ATC) and Flight Information Services (FIS).

ATCs are responsible for separation while FIS "provides information and assistance useful for the safe and efficient conduct of the flight"[11]. Improvements to ATM systems have been substantial; however the rate of technological advancements is failing to adequately match the growth of emerging fleets [24]. With additional pressure enforced by environmental bodies to reduce fuel consumption, developing an efficient and sustainable ATM system is imperative.
Arrival is a critical element in an aircraft’s trajectory and is subject to strict arrival procedures, runway availability, weather, curfew requirements and capacity limits. During high demand scenarios coordinating oncoming aircraft into a landing sequence while ensuring separation becomes very challenging. Aircraft arriving too early or too late cause disruptions for ATCs and other aircraft arriving on time, with respect to pre-allocated arrival times, as of the fluctuations in demand rather than an ordered spread. During peak hours, the struggle between demand and available runway capacity increases the strain on ATCs and causes delays. A key problem addressed within this paper is the relationship between aircraft on the ground, subject to ground delay programs, and aircraft that are airborne.

Scheduling flights for arrival in the attempt to minimise the cost of delays was investigated in [2]. Since then, research within the area of organising arrival schedules has been consistent and on-going through to current research projects. Odoni presented a deterministic model approach towards Single Airport Ground Holding Problems (SAGHP). This research was followed by Ball who investigated a deterministic network flow model that replaced deterministic demand with stochastic demand [19]. Based on mathematical proofs, Ball predicted improvements to ground holding problems and the levels of uncertainty associated with airport arrival capacity. This is achieved through the incorporation of a stochastic integer model placed under static demand. Following this, Hu conducted a study on dynamic arrival scheduling and sequencing aimed at improving safety, capacity and efficiency of operating airports [28].

The study of this paper looks at a four dimensional multi-criteria optimisation problem with the objectives of achieving minimum overall fuel consumption and establishing a more organised airspace with respect to departure and arrival schedules. The multiple objectives will be defined as minimum overall fuel consumption and on-time arrivals. To satisfy both objectives, not every flight route will demonstrate an optimal fuel path or a punctual arrival time. It is expected to see no fuel consumption savings for some trajectories and either late or early arrivals for others. It is expected that the combination of slightly non-optimal solutions, for individual flights, will generate a combined overall optimal result.

A deterministic model has no random factors with the output determined by defined parameters. This is in contrast to a stochastic model which inherits some randomness in the elements or description of the model. Stochastic processes involve probability, making it a suitable approach for trajectory planning and management. A critical factor in a stochastic environment, as air travel is, is how to incorporate unforeseen events. These uncertainties are often the source of disruption with a flow-on effect to all flights.

In order to address these uncertainties, with respect to both flow management and trajectories, the model will be subject to dynamic re-optimisations every set time period. These re-optimisations will be subject to updated information including, but not limited to, weather and traffic flow data (Fig. 1). In addition to ‘new’ information, the model will also consider ‘old’ data from earlier on the trajectory in its decision making. ‘Old’ information refers to information/data used within the last re-optimisation. ‘New’ information refers to the updated information/data provided for the next re-optimisation. By considering both old and new data, changes in circumstances can be modelled. It is anticipated that this will allow for improved decision making in the presence of uncertain information, specifically in long range flow.
This paper provides an extensive background on the problems addressed. It will then cover the mathematical methodology for the problem and go on to explain further work to be conducted within this area of research.

2 DELAYS

As per current ATFM, a number of different approaches are taken for delaying aircraft in the event that a delay is required.

2.1 Ground Delay

Ground delays involve delaying an aircraft queued for departure. This delay may be issued in response to poor weather limiting capacity or to manage demand and capacity in the air and at arrival airports. This option is cost effective in terms of fuel compared to absorbing delay while airborne; however it can cause problems in terms of runway occupancy. The gate cannot remain occupied if there are other aircraft scheduled to depart.

2.2 Airborne Delay

Airborne delays can be applied through a few different options. Determining which option to utilise will depend on the scenario and how much delay is required.

2.2.1 Holding

‘Holding’ patterns are used to delay an airborne aircraft, while keeping it within a specified airspace. This method is usually taken when the aircraft has arrived at its destination airport but cannot land due to traffic congestion or poor weather. Also, at some high traffic airports around the world, holding is often used as part overall arrival sequencing strategy. This ‘flying in circles’ approach is inefficient and costly as it inherits increased fuel consumption, increases running costs and involves occupying operational airspace, thereby increasing controller workload.

2.2.2 Speed and Altitude Control

Delays can also be absorbed through controlling an aircraft’s speed and altitude while en-route. Increasing the speed will increase the Mach number, \( M \), of the aircraft. Flying at a higher altitude will decrease the speed of sound \( a \), due to the reduction in temperature, \( T \), and therefore decrease the airspeed.
\[ a = \sqrt{\gamma R T} \]  
\[ V = M \sqrt{a} \]  

(1)  
(2)  

\( R \) is the universal gas constant for air \((287.04 \ \text{m}^2/\text{Ks}^2)\)  
\( \gamma \) is the isentropic expansion coefficient for air \((1.4)\)

This particular method proves difficult for long range flights due to their inherit uncertainty. More accurate predictions are made once the aircraft is closer to its destination. By this point, there may not be enough opportunity to utilise speed and altitude changes for delay absorption.

### 2.2.3 Vectoring

Vectoring involves ATCs assuming navigation of the aircraft. This can be used for distributing delays by diverting the aircraft from its original path. Similar to airborne holding, this approach is also costly in terms of both fuel and running costs as the aircraft is operating over a longer time period.

The choice of which delay to employ is very conditional. It will depend on a number of factors including traffic flow conditions, aircraft location and airspace design. As a result, not all delay strategies are appropriate for both short and long haul flights. For example, speed control would most likely not be applied to a short haul domestic flight, but rather, it would be subject to a pre-departure ground delay instead. Long haul flights, on the other hand, might not be subjected to a ground delay as they might depart from outside the jurisdiction area of a certain ATFM provider. Table 1 summarises which delays are more appropriate for short and long haul flights.

### Table 1: Delays for short and long haul flights

<table>
<thead>
<tr>
<th>Delay</th>
<th>Short Haul Flights</th>
<th>Long Haul Flights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Delay</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Holding</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Speed &amp; Altitude Control</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Vectoring</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

An alternative option for delay absorption is the utilisation of prevailing winds. Similar to the approach of speed and altitude control, these winds can be used to both speed up and slow down an aircraft whilst en-route. Currently, wind patterns are implemented into aircraft trajectories, mainly for the purpose of improving flight efficiency. An example is Australia’s Flextracks program which involves publishing wind data for airlines flying into and out of Australian airspace, allowing them to utilise favourable wind patterns [1]. This approach aims at reducing fuel consumption and flight time.

Within this study, winds will be incorporated as a means of strategic delay absorption. Similar to speed and altitude control, this option would be more suitable for long haul flights. The wind speed, \( W_S \), will affect the ground speed of the aircraft. It will either increase or decrease the groundspeed, with respect to the aispsed, depending on the direction of the wind relative to the heading of the aircraft.

\[ V = M \sqrt{\gamma R T} + W_S \]  

(3)
3 DYNAMIC RE-OPTIMISATION

Developments in ATFM within literature involve the study of Collaborative Decision Making (CDM) which has the potential to increase the accuracy of decision making during an aircraft's trajectory [19]. The concept of CDM involves sharing information across various stakeholders and thereby granting ‘decision making responsibility’ to more than one party. The purpose of shared information is also to create ‘common situational awareness’ for both air traffic managers and airspace users, alleviating the load on air traffic controllers by allowing for collaboration with pilots [18]. This approach diverges away from the conventional procedure of having Air Traffic Controllers (ATCs) and traffic flow managers as the fundamental decision makers [18].

Weather conditions and unforeseen events are a few of many contributing factors to trajectory and traffic flow uncertainty. Uncertainty in a flight route reduces decision making time frames, as evident in delayed arrival procedures. Increasing available information throughout the duration of a flight route will allow for decision making at an earlier time in the trajectory, specifically for long range flight routes. Additionally, it is difficult to predict what traffic flow scenarios will be for long range flights when they are closer to their destinations. Specifically, a problem facing current ATFM is the effect that aircraft on the ground have on those that are airborne. If a long range flight is airborne and heading for its destination and a shorter range flight is preparing to depart to the same destination, who do you delay in the event that a delay is required? Generally, the long range aircraft takes precedence. If the airspace is full, ATCs will attempt to avoid introducing more aircraft into that airspace. Additionally, the aircraft queued for departure does not inherit a fuel penalty when subjected to a ground delay. The problem with this scenario is two-fold. Firstly, while the shorter range aircraft is not inheriting a fuel penalty, it is being subjected to a time delay that ultimately increases cost and disrupts passengers. Secondly, this delay may result in reactionary delays to other aircraft as it will need to be re-allocated departure and arrival times. Frequently updated information such as weather data, traffic conditions and updated airport schedules has the potential to prepare aircraft for arrival at an earlier stage in the trajectory.

Building on the ‘information sharing’ concept of CDM, dynamic re-optimisations are incorporated within the proposed optimisation model. The system re-runs every set time period and makes decisions based on information available at that time, including weather, traffic conditions and airport schedules as well as information utilised within previous re-optimisations. The key difference in these re-optimisations is that uncertainty is modelled and decisions are made based on the probability of circumstances. For example, if a long range aircraft needs to be delayed by 25 minutes, the system will model the uncertainty in that information. Instead of immediately trying to absorb the whole 25 minutes, it may only choose to slow the aircraft down by 5 to 10 minutes and re-address the situation at the next re-optimisation. Doing this will avoid scenarios in which the aircraft is slowed down then sped up again once new information dictates that the delay was unnecessary. Fig. 2 illustrates the changes in Allocated Time of Arrival (ATA) and Estimated Time of Arrival (ETA) that can occur over time. It also demonstrates the reduction in the level of uncertainty as the aircraft nears its destination.
The decisions made at each re-optimisation will be made for the entire trajectory as opposed to a section-by-section approach. It is anticipated that a larger look-ahead time will improve overall decision making (Fig. 3). The purpose of the incorporated dynamic re-optimisations is to make use of uncertain information present in aircraft trajectories. This will allow for decision making at earlier times in the trajectory, improving the accuracy of these decisions and reducing the likelihood of delays.

The inclusion of dynamic re-optimisations will not completely eliminate uncertainty within the model. It will limit the amount of ‘short notice’ scenarios and establish a larger time frame for decision making. More accurate decisions can be made when more information is available. However, due to the stochastic nature of ATFM, unpredictable scenarios will still be present.

Literature which looks at ‘updated information’ within ATFM includes [7][9][12][13]. Additionally, a study comparing the benefits and setbacks of dynamic and static optimisation can be found in [25]. The similarity between these papers is the incorporation of updated information to aid and improve decision making. What sets this study apart is the approach taken towards dynamic re-optimisation.
4 METHODOLOGY

4.1 Airport Scheduling

Similar to the simplified approach taken for conflict avoidance, airport scheduling for departure and arrival is also applied in a simplified manner with the assumption that ground delay programs are already in place. That is, pre-tactical (prior to departure) airport scheduling and organisation with respect to demand vs. available capacity is not investigated, but how it can be tactically achieved is (when aircraft are in flight). Detailed studies on alternative approaches to airport scheduling and ground delay programs can be found in [2][3][20][21][25][28].

$s_{df}^a$ denotes the scheduled departure time of flight $f$ from airport $a$ while $s_{lf}^a$ represents the scheduled landing time of flight $f$ from airport $a$. These scheduled times are based on a pre-allocated time table established for each airport. These constraints are applied within a relatively small margin so as not to jeopardise airspace capacity.

$$s_{df}^a \leq dm, \forall a \in A, f \in F$$

$$s_{lf}^a \leq \pm dm, \forall a \in A, f \in F$$

Within the optimisation, aircraft are not permitted to depart early. A defined margin, $dm$, is applied to allocated departure and arrival times allowing the aircraft to depart slightly late and arrive both slightly early and slightly later than scheduled. The defined margin is consistent across all aircraft and airports; however, it is reduced under busier traffic conditions to meet the higher demand. The time restrictions applied to arrival and departure slots are integrated as soft constraints. The optimiser will attempt to adhere to these constraints; however, other objectives may take priority and may result in the constraint not being met. Within this model, time is one of the objectives defined within the multi-objective function. The soft constraint approach relaxes the restrictions placed on pre-defined airport schedules, allowing time to be modelled as an objective as opposed to a hard constraint that must be met. The intention is to encourage aircraft to meet their designated slot times. Penalties are applied for diverting away from the scheduled time slot, as shown in Table 2.

Table 2: Departure and landing penalties

<table>
<thead>
<tr>
<th>Departure Penalty</th>
<th>Landing penalty</th>
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<tbody>
<tr>
<td>$q_{f,1} = ad_{df}^a - sd_{df}^a$</td>
<td>$q_{f,2} = al_{lf}^a - sl_{lf}^a$</td>
</tr>
<tr>
<td>$\varnothing(\lambda, q_{f,1}) = \min \lambda y_{f,1}$</td>
<td>$\varphi(\beta, q_{f,2}) = \min \beta y_{f,2}$</td>
</tr>
<tr>
<td>Subject to</td>
<td>Subject to</td>
</tr>
<tr>
<td>$y_{f,1} \geq 0$</td>
<td>$y_{f,2} \geq 0$</td>
</tr>
<tr>
<td>$q_{f,1} \leq y_{f,1} + dm$</td>
<td>$\pm(q_{f,2}) \leq y_{f,2} + dm$</td>
</tr>
</tbody>
</table>

Where $\lambda$ and $\beta$ are positive constants that control how strongly the constraints are enforced. The objective is to organise aircraft for arrival from the point of departure through to the last segment of the trajectory. This contrasts current procedures which only begin to organise arrivals during the last segment of the flight route (Fig. 4). This is a result of the level of uncertainty present in flight paths, specifically long range...
routes. It is difficult to predict the circumstances at arrival for a long range aircraft that has just departed. It is anticipated that the incorporation of dynamic re-optimisations will reduce, but not eliminate, this uncertainty.

Figure 4: Aircraft queued for arrival

Capacity limitations are placed on both departure and arrival airports. $D_a^t$ represents the capacity of airport $a$ at time $t$ for departing flights. $L_a^t$ represents the capacity of airport $a$ at time $t$ for landing flights and $f_a^t$ represents flight $f$ at airport $a$ at time $t$.

$$\sum (f_a^t - f_a^{t-1}) \leq D_a^t, \quad \forall a \in A, t \in T$$

$$\sum (f_a^t - f_a^{t-1}) \leq L_a^t, \quad \forall a \in A, t \in T$$

The above equations dictate that the number of flights departing or landing from a particular airport may not exceed departure or arrival capacity at that airport.

4.2 Conflict Avoidance

In addition to flow management considerations, separation between individual aircrafts is all acknowledged. To ensure realistic behaviour between neighbouring aircraft, a basic approach to conflict avoidance is applied. Detailed studies investigating alternative approaches to conflict avoidance can be found in [10][15][26][27]. Within the proposed model, hard constraints are defined for horizontal and vertical distances between aircraft, $H$ and $D$ respectively (Figs. 5 and 6).

Figure 5: Horizontal separation
In practise, separation standards and regulations are more detailed and cover a diverse range of scenarios. Within Australian airspace, these standards can be found in [8].

4.3 Objective Function

An optimal solution is one that satisfies the objective function subject to defined equality and inequality constraints. Within the context of this study the problem is defined as a multi-criteria optimisation problem with the objectives of time and fuel consumption. Time is represented by air and ground delays and the costs associated with these delays. In ATFM there are many aircraft and each has its own objective. In the final solution it is likely that a trade-off has to be achieved where all aircraft accept the compromise and are willing to co-operate. The definition of the objective has to be fair to all aircraft types, flights and operators.

\[
d_f = \text{Cost}_f^g \sum_{t \in T, a = P(f, A)} (a d_f^a - s d_f^a) \tag{8}
\]

\[
ad_f^a - s d_f^a \geq 0
\]

\[
l_f = \text{Cost}_f^g \sum_{t \in T, a = P(f, D)} \delta_f^a \tag{9}
\]

\[
\delta_f^a \geq a l_f^a - s l_f^a
\]

\[
\delta_f^a \geq 0
\]

\(\text{Cost}_f^g\) denotes the cost of holding flight ‘\(f\)’ on the ground for one unit of time. The total time units flight ‘\(f\)’ is held on the ground is determined from the actual departure time \(a d_f^a\) minus the scheduled departure time \(s d_f^a\). The same approach is taken for landing, with \(\text{Cost}_f^g\) representing the cost of holding flight ‘\(f\)’ in the air for one unit of time. The time unit is derived by subtracting the scheduled landing time \(s l_f^a\) from the actual landing \(a l_f^a\) time of flight ‘\(f\)’ at airport ‘\(a\)’. The final objective function is shown in equation 6, with \(FC_f\) denoting the fuel consumption of flight ‘\(f\)’.

Weights are allocated to each objective to signify the importance of one objective compared to the other. These weights will be modified with re-optimisations subject to expected arrival times, possible need for delay absorption and fuel consumption considerations. The optimality of a solution, whether it tends towards fuel or time, will depend on the weight of each objective. Secondary corrections to these weights
made subject to scenario based considerations, will allow for improved final solutions. The weighted and penalised objective function, which we minimise, is shown below.

\[
\max \left[ w_1 \left( \sum_{f \in F} \text{Cost}^a_f \sum_{t \in T, a = P(f, 1)} (ad^a_f) \right) - \frac{sd^a_f}{w_2 \left( \sum_{f \in F} \text{Cost}^a_f \sum_{t \in T, a = P(f, NS)} \delta^a_f \right)} \right], w_3 \left( FC_f \right), \left( \lambda \sum_{f \in F} \sum_{t \in T, a = P(f, 1)} H_e(y_{f, 1}) \right), \left( \beta \sum_{f \in F} \sum_{t \in T, a = P(f, NS)} H_e(y_{f, 2}) \right)
\]

The first term represents the cost associated with departure, the second term denotes the cost associated with landing and the third term represents the overall fuel consumption of all aircraft modelled. The fuel consumption is derived using aircraft performance data obtained from the BADA 3.6 user manual [23]. \( H_e \) is a smooth step function applied to the departure and landing penalty functions, the fourth and fifth terms respectively. This integrates an asymptotic trend to the penalties, after a particular time period, as opposed to a continuously increasing linear function (Fig. 7).

![Comparison between a linear function (left) and a smooth step function (right)](image)

**Figure 7: Comparison between a linear function (left) and a smooth step function (right)**

Penalties for not meeting scheduled time slots will vary depending on different scenarios and circumstances. To mirror this, \( H_e \) will change depending on traffic conditions. That is, for busier workloads \( H_e \) will increase and for lighter workloads, \( H_e \) will decrease.

The overall optimisation framework for the proposed multi-criteria optimisation model is shown in Fig. 8.
5 FUTURE WORK

The next step is to model the problem and generate results in support of the proposed methodology. One of the key problems addressed within this paper is the relationship between long and short range aircraft. Specifically, those long range aircraft that are airborne while the domestic aircraft is queued for departure. This problem will be addressed further in future work. Specific scenarios will be modelled in which shorter range aircraft queued for departure are disrupted by long range aircraft arriving too early or too late. It will be interesting to see if the model will favour the long range aircraft due to the fuel consumption term within the objective function or if it will attempt to find a balance in order to minimise the penalty costs associated with arrival and departure. The interesting aspect of a multi-criteria optimisation problem is observing which objective the model will lean towards under different scenarios and circumstances.

Building on from fairness between small and large aircraft, fairness between airlines will also be investigated. Aircraft will be grouped into airlines. Some of these airlines will consist of all domestic aircraft, some all international and some both. Constraints or penalties will be applied to ensure a level of fairness exists between these airlines. What will be interesting to note is the airlines that carry both small and large range aircraft. Will the model apply all delays to the shorter range aircraft queued for departure? This approach would still yield a low cost for that airline as the long range aircraft would not be inheriting any penalties.

Fuel consumption calculations are carried out based on performance data and performance equations specified within BADA 3.6 [23]. BADA 3.6 does have its limitations and the sensitivity of these limitations will be assessed with respect to the given problem. A comparison between BADA 3 and BADA 4 can be found in [23]. The research proposed within this paper addresses a flow management problem and not an individual aircraft performance problem. As a result, the sensitivity of the BADA 3.6 performance limitations is expected to be minimal.
6 CONCLUSION

Problems facing ATFM are large and complex, making it near impossible to address them all simultaneously. This paper looks at the problem of arrival scheduling from a specific viewpoint. It looks at the issues present between long and short range flights. Specifically, how they affect each other when one is airborne and the other is queued for departure. The distribution of delays between these aircraft is investigated, with future work intending to establish a ‘fair’ system between small and large operators.

Ground delays, airborne holds, vectoring and speed and altitude control are all delay absorption techniques of current ATFM procedures which are incorporated into the optimisation framework. An additional option implemented into the model is the use of prevailing winds for both slowing the aircraft down and speeding it up. This approach contrasts other methodologies which utilise high speed winds as a means of reducing fuel consumption [4][6][14][22].

Dynamic re-optimisations are incorporated with the intent of reducing the level of uncertainty present in trajectories, specifically long haul flights. The main difference with these re-optimisations is the inclusion of both past and updated information in decision making. Including ‘old’ data into decision making allows the model to assess how data and information has changed over time. Uncertainty is taken into consideration and decisions are then made based on the certainty of the relevant information.

Future work within this research will look at generating results and improving the proposed methodology based on these results. It is intended to propose a means for improving current approaches towards the problems addressed.

7 REFERENCE


