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UNSTRUCTURED SHOCK-FITTING CALCULATIONS OF TRANSONIC TURBO-MACHINERY FLOWS

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ABSTRACT
Even though shock-capturing techniques are the de-facto standard in the CFD simulation of turbo-machinery flows, the accurate estimation of shock-induced losses in transonic flows can be severely hindered by the numerical errors that are generated along a captured shock and convected downstream. Indeed, and despite their widespread use, shock-capturing techniques are known to be plagued by a number of drawbacks that are inherent to the numerical details of the shock-capturing process. In recent works, the authors have developed a novel *unstructured* shock fitting technique that has been applied to the computation of transonic, supersonic and hypersonic flows in both two and three space dimensions. The use of unstructured meshes allows to relieve most of the algorithmic difficulties that have contributed to the dismissal of the shock-fitting technique in the framework of structured meshes. In this paper, the proposed technique is applied to flows of turbo-machinery interest.

NOMENCLATURE
Φ inviscid flux balance
∞ free-stream
α angle of attack
C_p pressure coefficient
M Mach number
e cell index
i grid-point index
is isentropic

INTRODUCTION
CFD codes are nowadays being routinely used, even at the industrial level, not only for the analysis of existing turbo-machines, but also as one of the building blocks in the design and optimisation cycle. Modern CFD developers are either interested in making existing CFD codes faster by taking advantage of the emerging HPC architectures or in coupling existing CFD codes with other simulation tools to build up complex, multidisciplinary optimisation tools. By contrast, little attention is nowadays being paid to the numerical details of the discretization schemes available in state-of-the-art CFD codes which are often regarded as being mature enough to be blindly trusted. However, when simulating flows in turbo-machinery components operating at transonic speeds, it should be kept in mind that shock-capturing schemes, that represent the de-facto standard in all modern CFD codes, are plagued by a number of drawbacks that can severely hinder their predictive capabilities when discontinuities, such as shock waves or slip lines, are present. These anomalies include: excessive shock width, spurious oscillations arising along the captured discontinuity and the reduction of the order of accuracy within the entire shock-downstream region, to name just a few. Evidence has been found,

see e.g. Zaide and Roe (2011), that the troubles encountered with shock-capturing discretizations is intrinsic to the numerical details of the capturing process, in particular to the existence of intermediate shock points (located in between the pre- and post-shock states) that have little to do with the physical shock structure, but are a mere numerical artifact. It follows that a definite cure to the shock-capturing anomalies is unlikely to be ever found and remedies such as anisotropic mesh adaptation, beside having their own computational cost, can only alleviate some of the aforementioned drawbacks.

Over the last few years, the authors have started investigating the possibility of reviving an old technique, older than and alternative to shock-capturing, that can be used to model shock waves as well as other discontinuities. Shock-fitting dates back to the work by Emmons (1944) and consists in explicitely tracking the Lagrangian motion of shocks and contact discontinuities that are treated as "true" discontinuities that bound regions of the flow-field where a smooth solution to the governing PDEs exists. The technique has been made popular by Moretti and co-workers, see e.g. Moretti and Abbett (1966), at a time when CFD codes only used structured grids. The advent of fast computer architectures and some algorithmic difficulties, partly rooted in the use of structured grids, that prevented shock-fitting codes from being general-purpose, have contributed to the gradual dismissal of the shock-fitting technique, which is nowadays only being used (in the structured-grid framework) for selected applications, see e.g. Ma and Zhong (2003). The only publications known to the authors where shock-fitting (on structured grids) has been used for turbo-machinery application are: Hall and Crawley (1989); Xu and Ni (1989).

The use of unstructured grids, however, allows to relieve most of the algorithmic difficulties encountered when shock-fitting had been used in conjunction with structured meshes. This is the key contribution brought by the authors to the subject [see Paciorri and Bonfiglioli (2009); Ivanov et al. (2010); Paciorri and Bonfiglioli (2011); Bonfiglioli et al. (2013)] and in this paper we show that the *unstructured* shock-fitting technique can be used to compute transonic flows of turbo-machinery interest. This is accomplished by using one external and one internal flow test-case for which numerical and experimental results are available in the literature. Moreover, the advantages offered by the unstructured shock-fitting technique are highlighted by comparing shock-fitting and shock-capturing calculations on unstructured, triangular grids of nearly identical spatial resolution.

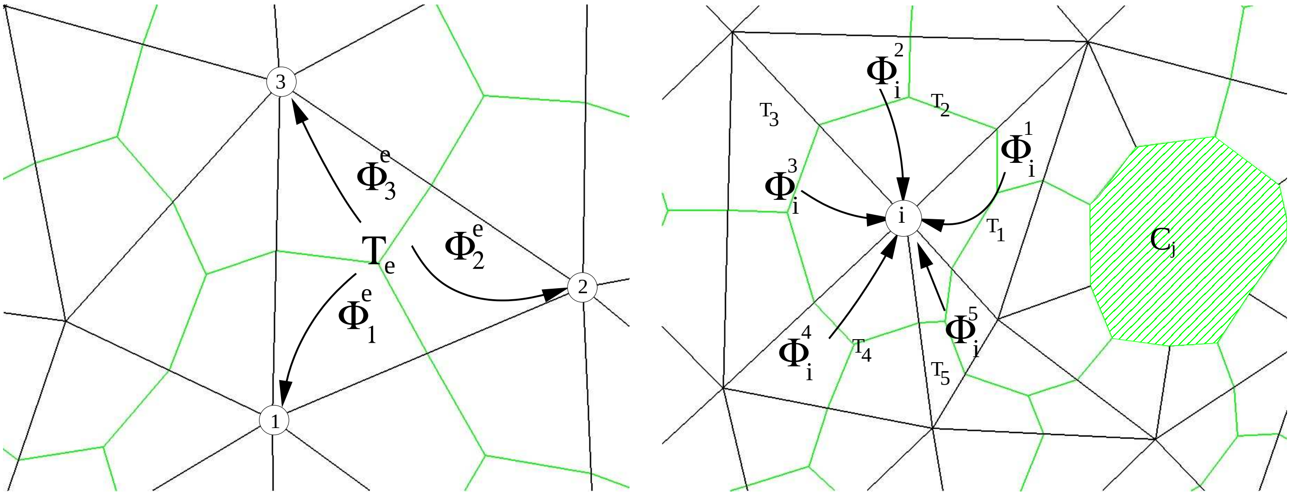
MATHEMATICAL AND COMPUTATIONAL MODELS

In this article attention is payed to the effects that different practices used to simulate shock-waves produce on the solution quality near the shocks and within those smooth regions of the flow field that are located downstream of the modelled shock-waves. Therefore, a mathematical model (the Euler equations) describing the dynamics of an inviscid, perfect gas, is deemed adequate for the purpose.

In the next two paragraphs the two approaches, namely shock-capturing and shock-fitting, that have been examined in this study will be briefly described. The shock-capturing discretization will be presented first, since it is also used in the shock-fitting approach to solve the governing PDEs in the smooth regions of the flow-field.

Shock-capturing

The `eulfs` code is an in-house, unstructured CFD solver that has been developed over the last twenty years; see Bonfiglioli (2000) for a detailed description of its basic features and Bonfiglioli and Paciorri (2013) for more recent developments. It relies on Fluctuation Splitting (FS), or Residual Distribution schemes, see Deconinck et al. (1993a); van der Weide et al. (1999); Abgrall (2006), for the spatial discretisation. In the FS approach the dependent variables are stored at the vertices of the computational mesh which is made up of triangles in the 2D space, and tetrahedra in 3D and are assumed to vary linearly and continuously in space. The inviscid flux balance Φ^e (also referred to as the cell residual or cell fluctuation) is evaluated over each triangular/tetrahedral element e by means of



(a) The flux balance of triangle e is scattered among its vertices.

(b) Grid-point i gathers the fractions of cell residuals from the surrounding cells.

Figure 1: Residual distribution concept.

a conservative linearisation Deconinck et al. (1993b) based on the parameter vector, see Roe (1981), and scattered to the element vertices using signals Φ_i^e, see Fig. 1(a). Within a cell e, the signals have to sum up to the net flux for conservation: $\sum_{i\in e}\Phi_i^e = \Phi^e$. The nodal residual is then assembled by collecting fractions Φ_i^e of the net fluxes Φ^e associated with all the elements by which the node i is surrounded, as schematically shown in Fig. 1(b). The various FS schemes proposed in the literature differ by the way cell residuals are split into signals. It is possible to construct schemes that depend linearly upon the solution (when solving a linear PDE) and are either monotonicity preserving, but limited to first order of accuracy, which is the case of the N scheme, or, if second order accurate, may lead to oscillatory behaviour in the neighbourhood of a captured discontinuity, which is the case of the LDA scheme. A non-linear scheme which captures the discontinuities monotonically and preserves second order of accuracy in smooth regions of the flow-field can be constructed by using a solution-dependent weighting function which blends the linear N and LDA schemes in such a way that the former scheme is activated only in the neighbourhood of the captured discontinuities whereas the latter is used elsewhere.

Shock-fitting

The unstructured shock-fitting algorithm that has been recently developed by the authors Paciorri and Bonfiglioli (2009, 2011); Bonfiglioli et al. (2013, 2014) consists of two key ingredients: *i*) a local re-meshing technique that constructs a time-dependent mesh in which the fitted discontinuities are internal boundaries of zero thickness and *ii*) an algorithm for solving the Rankine-Hugoniot jump relations that provides the Lagrangian velocity of the discontinuity and an updated set of dependent variables within the downstream side of the fitted shock. More precisely, in two space dimensions the fitted shock fronts are made of polygonal curves, i.e. a connected series of line segments (which we call the shock edges) that join the shock points. Two sets of flow states, corresponding to the upstream and downstream sides of the discontinuity, are assigned to each of the shock-points located on either side of the shock front. The downstream state and the shock speed are computed according to the Rankine-Hugoniot jump relations and the fitted shock is allowed to move throughout a background triangular mesh that covers the entire computational domain. At each time step, the shock line is inserted into the background mesh while ensuring that the edges that make up the shock front are also part of the triangular grid that covers the entire computational domain. The shock-insertion

algorithm proposed in our previous work, see Paciorri and Bonfiglioli (2009, 2011), has recently been reformulated and generalised by Zaide and Ollivier-Gooch (2014). The mesh modified by the shock-insertion algorithm is what we refer to as the "shock-fitting" grid, which differs from the background triangulation only in the neighbourhood of the shock front. The fitted shocks are treated as interior boundaries by the `eulfs` shock-capturing code described in the previous section which is used to solve the discretised governing equations in the smooth regions of the flow-field.

Numerical results

In this section we present numerical results for two different transonic flow configurations, one external and one internal.

The external flow test-case is primarily aimed at demonstrating that shock-fitting is capable of delivering accurate results even on very coarse grids. The internal flow case shows that the proposed unstructured shock-fitting algorithm is capable of dealing with the complex shock topologies that are encountered in turbo-machinery applications.

Both flow configurations examined here involve two-dimensional geometries. The three-dimensional version of the present unstructured shock-fitting algorithm is conceptually identical to its two-dimensional version described above and has already been applied, see Bonfiglioli et al. (2013), to the simulation of supersonic and hypersonic flows featuring isolated as well as interacting shocks. The application of the unstructured shock-fitting algorithm to three-dimensional turbo-machinery configurations is however challenging from the mesh-generation viewpoint, since it requires to handle the interaction of the fitted shock surfaces with the wetted surfaces of the turbo-machine. This is a feature which is not currently available in the 3D algorithm and, therefore, the applications presented herein are limited to two-dimensional configurations.

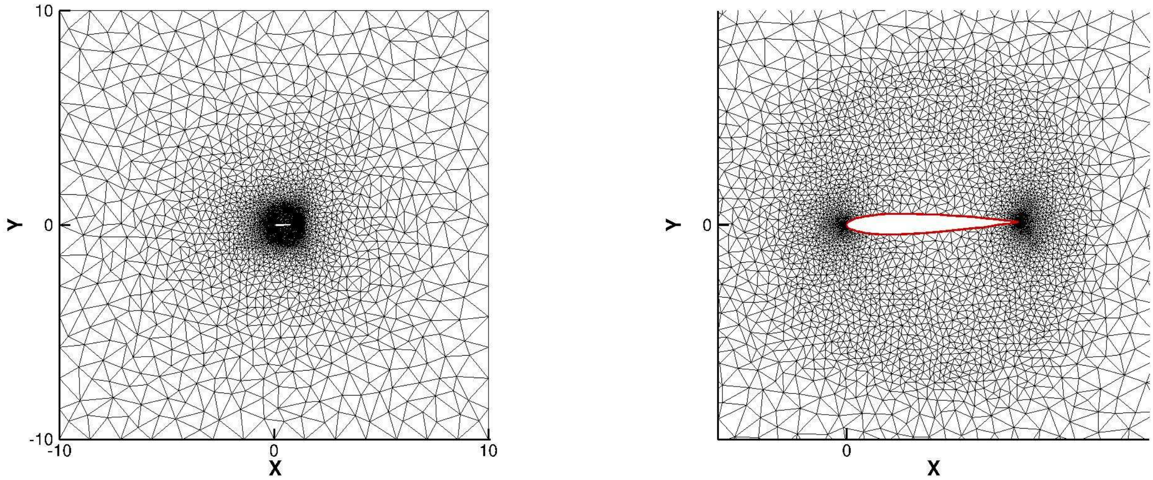
External, transonic flow past the NACA 0012 airfoil

The superior accuracy that fitted shock-waves deliver over captured ones is here illustrated by reference to a well documented external flow test-case, namely the two-dimensional, inviscid, transonic flow past the NACA 0012 airfoil at $\alpha_\infty = 0°$ degrees angle of incidence and free-stream Mach number equal to $M_\infty = 0.80$.

Reference solutions for this geometry and flow configuration have been obtained by Vassberg and Jameson (2010) with three well-known flow solvers (FLO82, OVERFLOW and CFL3D) using a family of 8 structured meshes with resolution up to 4096$\times$4096 control volumes in each coordinate direction, i.e. about 16 million cells, see Vassberg (2009). Such an extreme resolution has been chosen in Vassberg and Jameson (2010) in order to analyse the asymptotic convergence properties of different solvers on a specific inviscid, transonic problem. The reference solution we shall refer to in the following has been obtained using Jameson's FLO82 solver on the finest grid.

The unstructured shock-capturing calculation has been obtained using the grid shown in Fig. 2, which features 9912 triangles and 5024 mesh-points, 77 of which are placed along the airfoil's profile. The shock-fitting grid (which uses the shock-capturing mesh as background triangulation) is made of 9982 triangles and 5078 mesh-points.

A first comparison between the shock-capturing and shock-fitting solutions is shown in Fig. 3, where pressure iso-contour lines are displayed: it is evident that shock-fitting allows to obtain a much more realistic shock-thickness than shock-capturing, using grids of almost identical spatial resolution. The C_p distribution along the profile computed using unstructured shock-capturing and shock-fitting is compared with the reference solution in Fig. 4. Not only the unstructured shock-capturing calculation predicts an un-physically large shock-thickness, see Fig. 4(a), it also completely misses the so-called Zierep (2003) singularity that occurs at the foot of the shock which, by contrast, is picked-up by the unstructured shock-fitting and reference shock-capturing solutions. Figure 4(b) also reveals



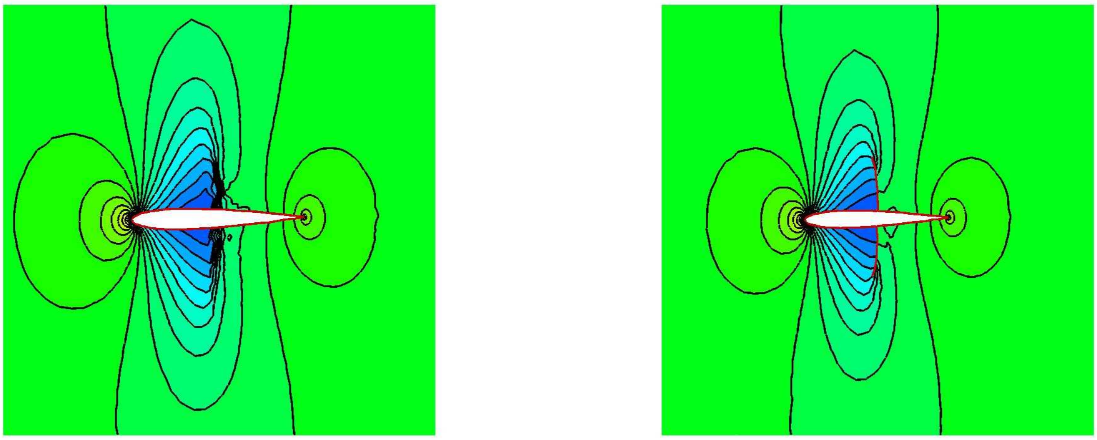
(a) Full computational domain. (b) Detail of the mesh around the profile.

Figure 2: NACA 0012 airfoil, α_∞ = 0°, M_∞ = 0.80: grid used in the shock-capturing calculation.

that the unstructured shock-fitting calculation is free from the un-physical pre- and post-shock oscillations that plague the reference shock-capturing calculation computed on a grid featuring three orders of magnitude more mesh-points. It also interesting to note, see Fig. 4(b), that the shock positions predicted by shock-fitting and the reference shock-capturing calculation differ by less than 0.005 chord units. This result shows that fitting, rather than capturing, shock waves significantly improves the performances of a gas-dynamic solver. Indeed, taking into account that using an efficient implementation, see Grottadaurea et al. (2011), the three-dimensional shock-fitting technique accounts for a small fraction ($10\% - 20\%$) of the overall iteration cost, its superior computational efficiency should be clear, in the sense that shock-fitting allows to obtain highly accurate solutions on (very) coarse meshes. This was precisely the reason behind the development of the shock-fitting technique based on structured meshes in the early CFD era. Shock-fitting was later abandoned when the improvements in computer speed made fine-grid solutions more affordable and the quest for general purpose codes clashed with a number of algorithmic complexities incurred by shock-fitting when used on structured meshes.

Internal, transonic flow past the VKI LS-59 GT rotor blade

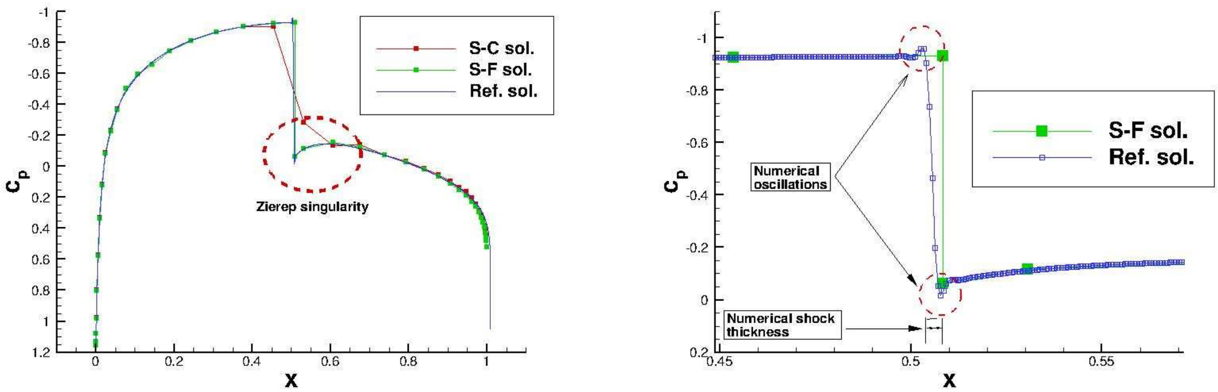
The Von Karman Institute gas turbine rotor blade (VKI LS-59) is a high loaded blade with a thick, rounded trailing edge, as shown in Fig. 5. This blade has been extensively tested, both experimentally by Kiock et al. (1986) and numerically, see e.g. Arnone et al. (1991); Arnone and Swanson (1993). The computational domain is a slice, extracted from an infinite cascade, that encloses only one blade. The upper and lower boundaries of the slice are drawn in such a way that the periodicity condition can be imposed on these two boundaries. Total temperature, pressure and flow angle are prescribed on the inlet boundary and static pressure on the outlet. The triangular mesh used in the present unstructured, shock-capturing calculation is shown in Fig. 5: it is made of 13093 grid-points and 25350 triangles; 256 points are placed along the profile of the blade. Computations relative to an isentropic exit Mach number of 1.2 are shown in Figs. 6 through 8. Figure 6 compares the density iso-contour lines computed in the present unstructured shock-capturing and shock-fitting calculations (shown, respectively, in Figs. 6(b) and 6(c)) with the result obtained by Arnone et al. (1991) using a 449$\times$17 non-periodic C-type structured grid.



(a) Shock-capturing solution.

(b) Shock-fitting solution.

Figure 3: NACA 0012 airfoil, α_∞ = 0°, M_∞ = 0.80: pressure iso-contour lines.



(a) Full profile.

(b) Detail of the shock-foot region.

Figure 4: NACA 0012 airfoil, α_∞ = 0°, M_∞ = 0.80: pressure coefficient distribution along the profile.

Concerning the two shock-capturing calculations, Figs. 6(a) and 6(b), there are noticeable differences in the shock structure: the shock reflected off the suction side is better captured in the unstructured calculation and also the trailing edge shock patterns are different. This is likely due to differences in the mesh resolution between the structured and unstructured grids. However, also the shock-capturing and shock-fitting calculations on the unstructured grid show noticeable differences in the trailing edge shock structure, even if the two triangular grids are nearly identical. The aforementioned differences can be better seen in Fig. 7, which shows an enlarged view of the density iso-contour lines in the trailing edge region for both unstructured-grid calculations: shock-capturing in Fig. 7(a) and shock-fitting in Fig. 7(b). The key difference between these two solutions is the much more pronounced unsteadiness of the shock-fitting solution, which suggests that it features a reduced level of numerical viscosity, compared to the shock-capturing one. The entire shock topology downstream of the trailing edge is consequently affected and it is therefore different in the two sets of unstructured-grid calculations. As far as flow unsteadiness is concerned, it should be clear from the algorithmic description, that the shock-fitting algorithm is inherently un-steady, since both

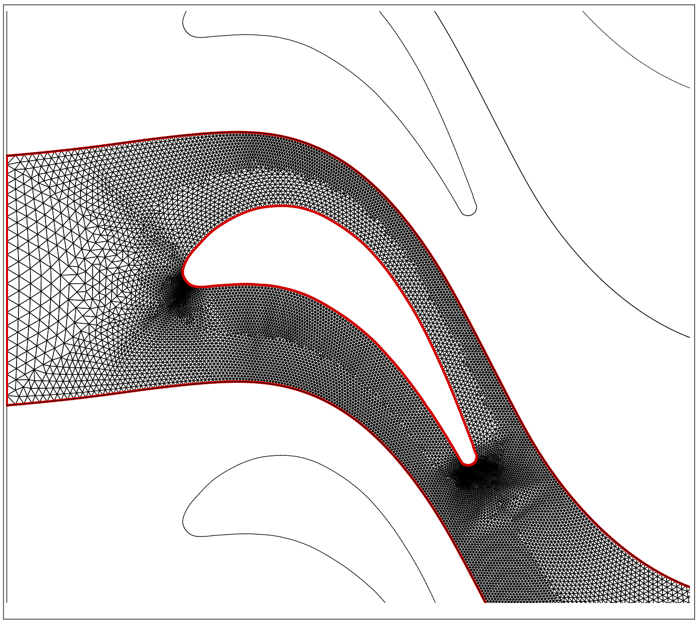


Figure 5: VKI LS-59 GT rotor blade: computational domain and unstructured triangular mesh.

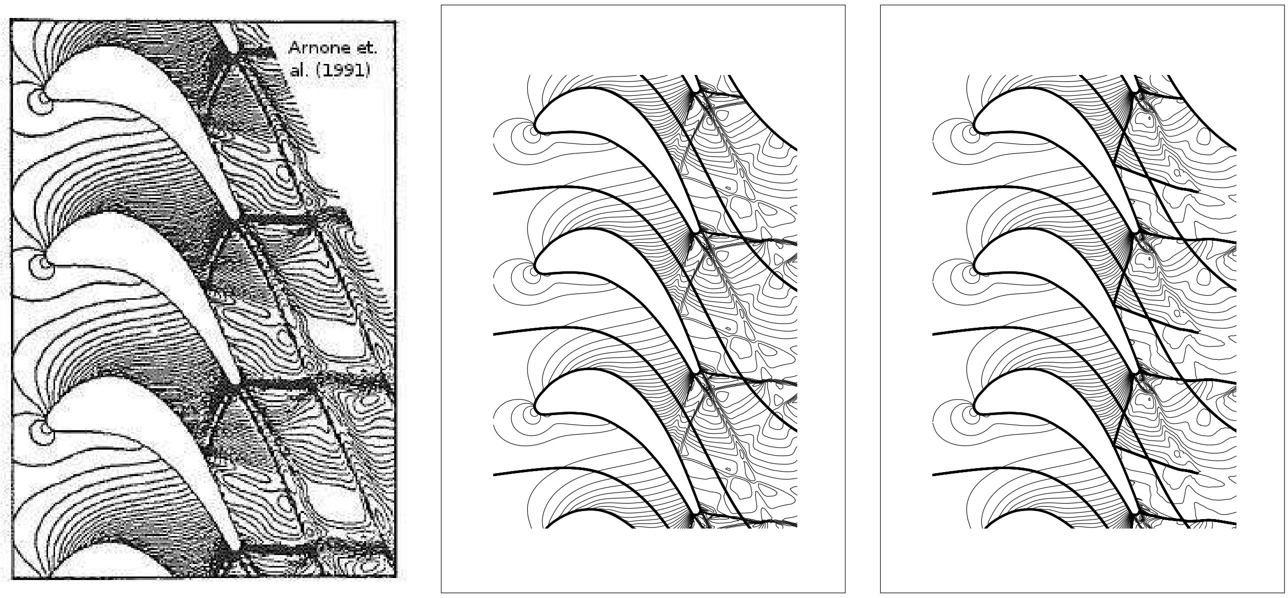
the mesh and solution change with (pseudo-)time. Whenever a steady solution to the discretised governing equations exists, the mesh will reach a stationary configuration and the residuals of the shock-capturing solver converge towards machine zero. This is clearly not the case for the present test-case, which exhibits a certain amount of unsteadiness even in the unstructured shock-capturing calculation. This is in contrast with the reference calculation, see Arnone et al. (1991), were convergence to machine zero is reported. The temporal accuracy of the shock-fitting calculation presented herein is limited to first order in time and a second-order-accurate version has only recently been developed, see Bonfiglioli et al. (2014).

It is also worth mentioning that those discontinuities that are not fitted in the shock-fitting calculation can however be captured thanks to the use of a shock-capturing discretization away from the fitted discontinuities. This is the case of the λ-shock structure that is visible in both sets of calculation shown in Fig. 7.

Finally, a more quantitative comparison among the two sets of unstructured-grid calculations, the reference structured-grid calculation and the available experimental data is given in Fig. 8, which shows the isentropic Mach number distribution along the blade. Figure 8(a), which has been reprinted from Arnone et al. (1991), shows the results of the reference calculation (Euler (no wedge) is the one to compare with) and Fig. 8(b) shows the unstructured shock-capturing and shock-fitting calculations compared against the experimental data. Not surprisingly, the two sets of unstructured-grid calculations give identical M_{is} distributions on the pressure side and, on the suction side, up to the point where the shock impinges on the blade; differences can be seen in the entire shock-downstream region and are clearly due to the two different shock modelling practices.

CONCLUSIONS

Despite the fact that shock-fitting has for long been regarded as an obsolete numerical approach, un-suitable to compute complex flows, our recent work has shown that fitting shocks on unstructured grids allows to relieve most of the algorithmic difficulties that have contributed to the dismissal of the shock-fitting technique in favour of the simpler shock-capturing paradigm. We hope that these results will stimulate a revision of some common beliefs about the shock-fitting approach.

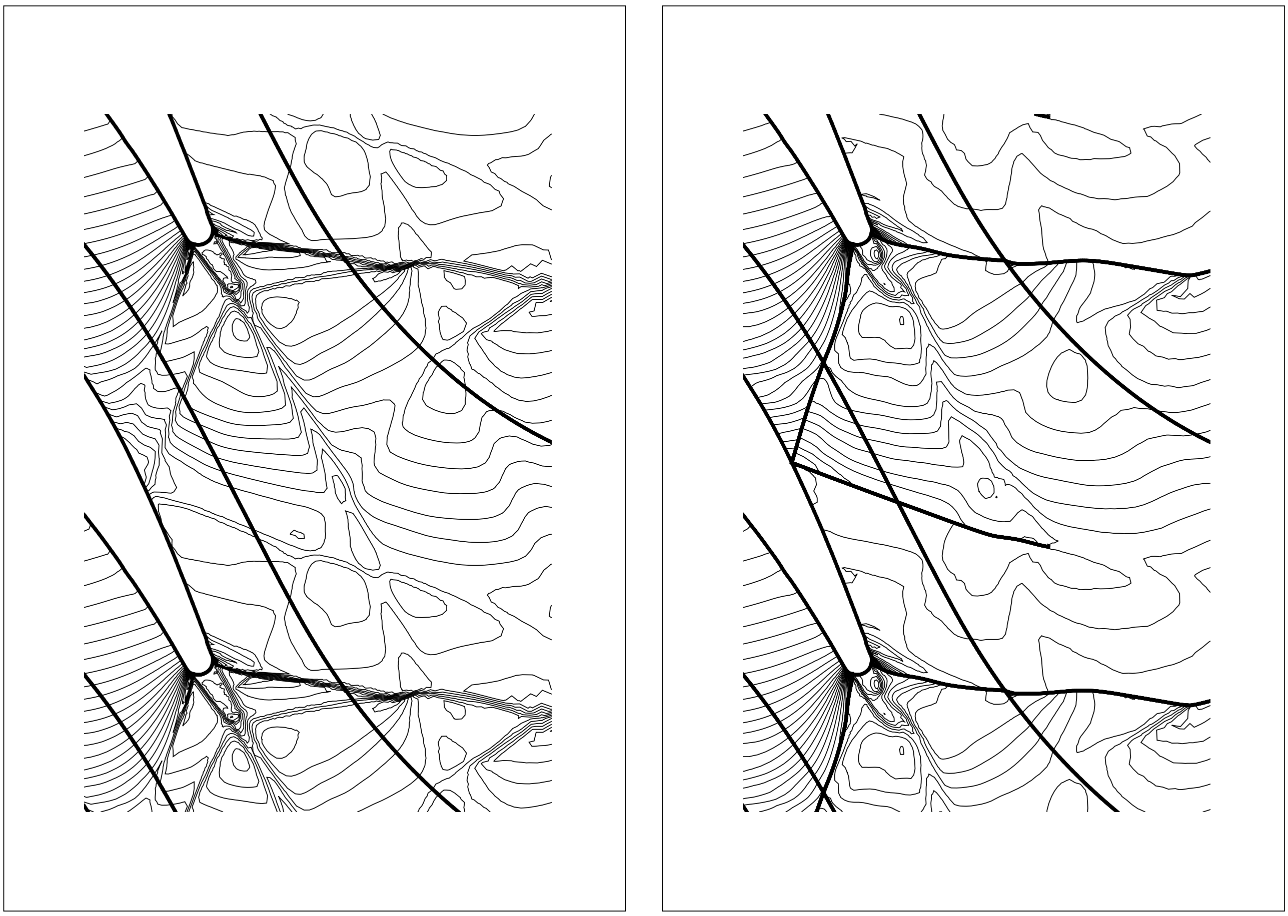


(a) Re-printed from Arnone et al. (1991).

(b) Present shock-capturing calculation.

(c) Present shock-fitting calculation.

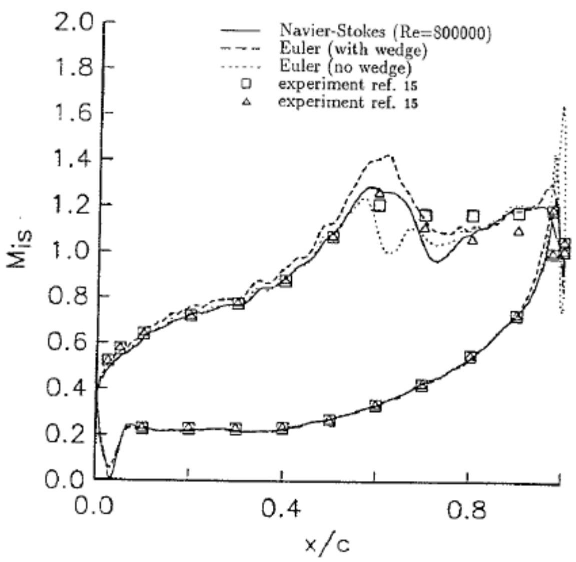
Figure 6: VKI LS-59 GT rotor blade ($M_{is} = 1.2$): density iso-contours.



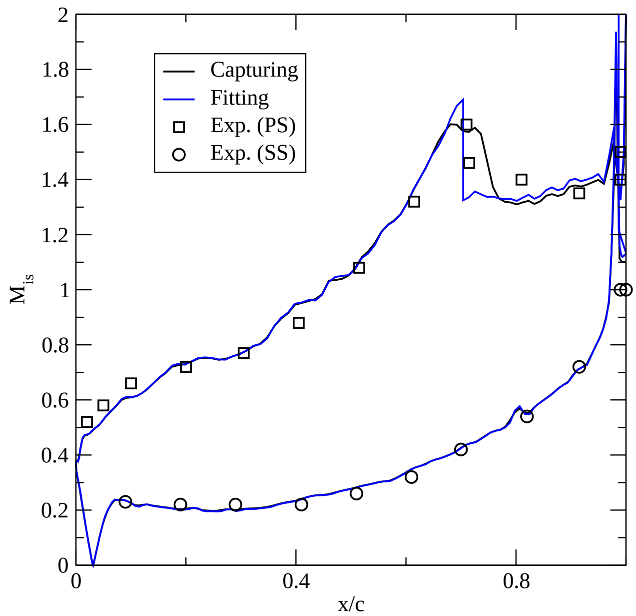
(a) Shock-capturing.

(b) Shock-fitting.

Figure 7: VKI LS-59 GT rotor blade ($M_{is} = 1.2$): density iso-contours revealing the shock structure in the trailing edge region; unstructured grid calculations.



(a) Re-printed from Arnone et al. (1991).



(b) Present calculations.

Figure 8: VKI LS-59 GT rotor blade ($M_{is} = 1.2$): isentropic Mach number distribution along the blade.

In this paper, in particular, we have not only shown that unstructured shock-fitting retains the most notable property also possessed by the shock-fitting technique originally developed for structured mesh, i.e. remarkable accuracy even on coarse discretizations, but also that unstructured shock-fitting allows to deal with the complex shock topologies that occur in transonic turbo-machinery flows much more appropriately than it was possible with structured grids. It is also important to underline that the unstructured shock-fitting approach is highly modular, in the sense that it can be interfaced with different mesh generators and (shock-capturing) gas-dynamic solvers, all being treated as black boxes. Flexibility has been gained thanks to the shift from the structured-grid framework traditionally used with shock-fitting methods to the unstructured-grid one.

It is also evident, however, that supplementary features are needed for the proposed technique to be applicable to real life turbo-machinery applications; these include: viscous and three-dimensional effects. The three-dimensional version of the proposed unstructured shock-fitting algorithm has already been developed and published by the authors and has no conceptual differences with respect to its two-dimensional version that has been illustrated in the present paper. The difficulties posed by the addition of the third spatial dimension have to do with mesh generation along and around the fitted shock surfaces and the handling of those shock surfaces that interact with solid walls or other shock surfaces. These are, however, issues that are not specific to shock-fitting, but to mesh generation. A fruitful collaboration with colleagues working on mesh generations is presently ongoing, and we therefore believe that these difficulties will be mitigated or even completely overcame by developing ad-hoc mesh generation tools. Even though not addressed by the authors, the interaction between a fitted shock and a boundary-layer has already been dealt with in the structured-grid framework so that its generalisation to unstructured grids should pose no major problem. Therefore, we are confident that the use of the proposed unstructured shock-fitting approach might become, in a reasonable time scale, a viable option for the numerical simulation of turbo-machinery flows.

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