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Linear Parameter Varying Control of an Agile Missile Model Based on the Induced L_2-norm Framework

Raziye Tekin and Harald Pfifer

Abstract This paper deals with the application of a linear parameter varying (LPV) controller synthesis for a modern air defense missile model. The model represents a challenging control problem due to the wide operation range. First, an LPV model of the missile is constructed via a novel approach of function substitution. Then, an LPV controller is designed based on the induced L_2-norm framework. A mixed sensitivity weighting scheme is applied to specify the performance requirements. In order to fulfill various time and frequency domain criteria, a multiobjective optimization is used to tune the weighting functions of the mixed sensitivity weighting scheme. Finally, the robustness and performance of the controller is evaluated by nonlinear simulations.

1 Introduction

Tactical missiles operate over a large flight envelope. Moreover, they need to be able to perform rapid maneuvers leading to fast variations in the flight conditions. Hence, a major requirement on the control system is to be able to retain good performance despite these fast varying parameters. An autopilot designed on a set of linear models over different flight conditions and ad hoc scheduling seems unsuitable to fulfill this demand. Such a design always assumes sufficiently slow parameter variations. In contrast to this classical approach, the LPV framework introduced in [1] can directly deal with fast varying scheduling parameters.

In this paper, first a brief theoretical background is given including the derivation of LPV systems and the controller synthesis in the induced L_2-norm framework. Then, the notions of generalized plant and mixed sensitivity weighting schemes are introduced. Afterwards, an LPV model is obtained for the considered nonlinear

Raziye Tekin and Harald Pfifer
Institute of System Dynamics and Control, German Aerospace Center - DLR, Muenchner Str. 20, 82234 Wessling, Germany e-mail: raziye.tekin@dlr.de - e-mail: harald.pfifer@dlr.de

missile based on a function substitution proposed in [2]. A comparison of classical Jacobian linearization approach to obtain LPV model and function substitution is given to show the effectiveness of the latter method. A state feedback LPV controller is designed for the function substitution based LPV model using the methods proposed in [1] to track the normal acceleration commands. The controller objectives are defined in the frequency and time domain. In order to fulfill the requirements, the weightings are parameterized related to the system dynamics regarding the closed loop behavior of the LPV system. A multiobjective optimization is used to tune the parameters of the weightings. Finally, the performance and robustness of the resulting LPV controller is assessed using nonlinear simulations.

2 Theoretical Background

In this section, linear parameter systems and methods to obtain LPV systems are introduced. Then, a brief overview of the concept of generalized plant is presented. Finally, the solution to the state feedback LPV synthesis in the L_2-norm framework is given.

2.1 LPV Systems

LPV systems are defined as systems which are linear in $[x^T \ u^T]^T$ but nonlinear in some exogenous time varying parameters $\rho(t) : \mathscr{R}^+ \rightarrow \mathscr{P}$ as shown in Eq. 1.

$$\begin{bmatrix} \dot{x} \\ \hline y \end{bmatrix} = \left[\begin{array}{c|c} A(\rho(t)) & B(\rho(t)) \\ \hline C(\rho(t)) & D(\rho(t)) \end{array}\right] \begin{bmatrix} x \\ \hline u \end{bmatrix} \tag{1}$$

For most physical applications, the parameter variation rate is bounded i.e. $\dot{\rho}(t) : \mathscr{R}^+ \rightarrow \dot{\mathscr{P}}$ with

$$\dot{\mathscr{P}} := \{q \in \mathscr{R}^{n_p} \mid |q_i| < \nu_i \ , \ i = 1, \ldots, n_p\} \tag{2}$$

It shall be pointed out that an LPV system reduces to an LTI (linear time invariant) system if ρ is constant and it reduces to an LTV (linear time varying) system when ρ is along a predefined trajectory. In contrast to LTV systems, the parameter trajectory is not now a priori but assuming to be online measurable for LPV systems. Hence, the following synthesis is not performed along a trajectory $\rho(t)$ with $\dot{\rho}(t)$ but over the corresponding parameter spaces represented by $p \in \mathscr{P}$ with $q \in \dot{\mathscr{P}}$ as given in Eq. 3.

$$\begin{bmatrix} \dot{x} \\ \hline y \end{bmatrix} = \left[\begin{array}{c|c} A(p) & B(p) \\ \hline C(p) & D(p) \end{array}\right] \begin{bmatrix} x \\ \hline u \end{bmatrix} \tag{3}$$

2.1.1 Derivation of LPV Systems

There are various methods in literature to obtain LPV systems from nonlinear systems such as Jacobian linearization, function substitution and state transformation. Detailed information about derivation of LPV systems can be found in [3]. In this section, only the two methods which are applied in this study are introduced.

Jacobian Linearization: This method is the most common methodology used to obtain LPV systems. It requires trimming at a set of equilibrium points and linearizing at equilibrium points which must span the operation space of the system. There are various examples using Jacobian linearization to derive LPV systems and design LPV controllers, e.g [4]. While it is the most widespread, it is generally not possible to acquire the transient behavior of the nonlinear system. See [5] for the shortcomings of this approach.

Function Substitution: This approach has been first presented in [6] and further enhanced in [2] and [3]. Unlike the Jacobian linearization approach, this method does not rely on a set of equilibrium points. It is essentially only an analytic transformation of the nonlinear differential equations. Hence, it is possible to capture the transient behavior of a nonlinear system well. Details of the chosen analytic transformation for the missile model are presented in Section 3.1.

2.2 LPV Controller Synthesis

In the presented LPV controller synthesis, the requirements of the closed loop system are specified using induced L_2-norm (i.e. input/output gain) performance objectives. For more detailed description of the method see [1].

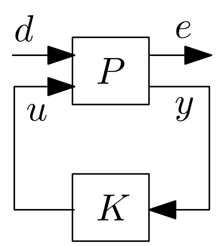


Fig. 1 Generalized plant

In Fig. 1, a generalized plant is presented where P represents the LPV plant including weightings and K the controller. The synthesis problem is to find a controller that minimizes the closed loop induced L_2 gain γ from the performance inputs d to performance outputs e as described below:

$$\min_K \|\mathscr{F}_l(P,K)\|_{\mathscr{L}_2\to\mathscr{L}_2} \tag{4}$$

s.t. $\mathscr{F}_l(P,K)$ is stable for all admissible trajectories $\rho(t)$ with bounded parameter variation rate $\dot{\rho}(t)$. $\mathscr{F}_l$ describes the lower fractional transformation, as seen in Fig. 1.

The synthesis problem Eq. 4 can be solved by applying a generalized version of the Bounded Real Lemma for LPV systems. The Bounded Real Lemma provides an upper bound on the induced L_2-norm of a given LPV system for the bounded parameter variation rate $\dot{\rho(t)}$. In order to shorten the following notation, a differential operator $\partial X(p,q)$ is introduced as in [7]. For continuously differentiable $X(p)$, the differential operator $\partial X(p,q)$ is defined as $\partial X(p,q)$=$\sum_i^{n_p} \frac{\partial X(p)}{\partial p_i} q_i$. With this choice of $\partial X(p,q)$ represents the first time derivative of $X(\rho(t))$ for any trajectory $\rho(t)$.

Bounded Real Lemma: Let P be an LPV system of the form Eq. 3 then P is exponentially stable and $\|P\|_{\mathscr{L}_2 \to \mathscr{L}_2} < \gamma$ for all $\forall(p,q) \in \mathscr{P} \times \dot{\mathscr{P}}$, if $\exists X(p) > 0$ such that $\forall(p,q) \in \mathscr{P} \times \dot{\mathscr{P}}$

$$\begin{bmatrix} A(p)X^T(p) + X(p)A^T(p) + \partial X(p,q) & X(p)B(p) \\ B^T(p)X(p) & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C^T(p) \\ D^T(p) \end{bmatrix} \begin{bmatrix} C(p) & D(p) \end{bmatrix} < 0 \tag{5}$$

2.2.1 State Feedback Synthesis

With the help of the Bounded Real Lemma, the synthesis problem defined in Eq. 4 can be turned into a semidefinite program. In this study, only the state feedback problem is considered, as the missile example belongs to this class. In this case, the generalized plant can be written as Eq. 6.

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(p) & B_1(p) & B_2(p) \\ C_{11}(p) & 0 & 0 \\ C_{12}(p) & 0 & I \\ I & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \tag{6}$$

Introducing new variables $R(p) = \gamma^{-2}X(p)$, $\partial R(p,q) = -\gamma^{-2}X^{-1}\partial X(p,q)X^{-1}(p)$ and $\hat{A}(p) = A(p) - B_2(p)C_{12}(p)$, and applying the Bounded Real Lemma on the closed loop $F_l(P,K)$, the controller synthesis problem Eq. 4 becomes:

$$\min_{R(p)} \gamma, \text{ s.t. } \exists(p,q) \in \mathscr{P} \times \dot{\mathscr{P}}$$

$$R(p) > 0$$

$$\begin{bmatrix} R(p)\hat{A}(p)^T + \hat{A}(p)R(p) - \partial R(p,q) - B_2(p)B_2^T(p) & R(p)C_{11}^T(p) & \gamma^{-1}B_1(p) \\ C_{11}(p)R(p) & -I & 0 \\ \gamma^{-1}B_1(p)^T & 0 & -I \end{bmatrix} < 0 \tag{7}$$

It is not possible to solve Eq. 7 over the whole function space of $R(p)$. Hence, $R(p)$ has to be restricted to a finite dimensional space. The function $R(p)$ is defined by a

set of basis function $g(p)$ of the form:

$$R(p) = \sum_j g_j(p) R_j \tag{8}$$

Note that the constraints in Eq. 7 are actually infinite dimensional due to their dependency on p and q. Since q enters in Eq. 7 only affinely and the set $\dot{\mathscr{P}}$ is a polytype, it is sufficient to check the constraints on the vertices of $\dot{\mathscr{P}}$. To deal with dependency on p, a grid over P is generated and the constraints are only checked on this grid. This approach is common in literature, see for example [1]. Finally, the state feedback control law is given by

$$K(p) = -(B_2^T(p) R^{-1}(p) + C_{12}(p)), \tag{9}$$

where the controller $K(p)$ is a continuous matrix function of p.

2.2.2 Generalized LPV Plant

The mixed sensitivity weighting scheme is a well known and common concept to define the controller objectives in the induced L_2-norm framework [8]. The generalized plant which is the plant augmented by the weighting functions is depicted in Fig. 2. In this weighting scheme, the closed loop sensitivity function $S = (I + GK)^{-1}$ and KS are shaped by W_y and W_u respectively. Due to the choice of the performance input d as seen in Fig. 2, the sensitivity function S and KS are weighted by the plant.

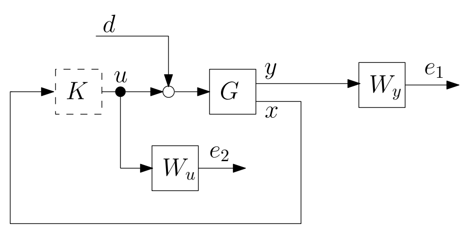


Fig. 2 Structure of mixed sensitivity scheme for generalized plant

The transfer functions of W_u and W_y are given in Eq. 10 where ω_{ro} is the desired roll-off frequency of the controller input, k_u the low frequency gain of W_u, ω_b is the desired minimum bandwidth of the closed system and k_y is high frequency gain of W_y to penalize the overshoot which are further going to be optimized for LPV controller design.

$$W_u = 100 k_u \frac{s + \omega_{ro}}{s + 100\omega_{ro}}, \quad W_y = k_y \frac{s + \omega_b}{s + 0.005} \tag{10}$$

3 LPV Controller Design for the Missile Model

In the following section, the design process of the LPV controller is described for longitudinal motion of a missile. First, an LPV model of the nonlinear missile dynamics is derived by function substitution. Jacobian linearized and function substitution based LPV models are compared with nonlinear simulation. A mixed sensitivity weighting scheme is applied for the controller design. After defining the structure of the weighting functions given in Section 2.2.2, they are optimized. Then, an LPV controller is designed based on the approach described in Section 2.2.

3.1 LPV Modeling

The model and aerodynamic database is taken from [9] which includes all flight regime including boost phase with thrust vector control. However, in this paper, only the post boost phase is studied. The body axes longitudinal motion of the tail controlled air defense missile, extracting flexible and high angle of attacks phenomenas, can be described by the following differential equations:

$$\dot{w} = \frac{1}{m}(F_z - qu), \quad \dot{q} = \frac{M}{I_y}, \tag{11}$$

where m is the missile mass, I_y the inertia in principal axes, q the pitch rate, α the angle of attack and u the longitudinal velocity. Since in this study only the post boost phase is studied, mass, inertia and center of gravity are constant and the gravitational force is neglected. All the considered forces and moments are aerodynamic. The related aerodynamic forces and moments are modeled as:

$$\begin{aligned} F_z &= Q(Ma,h)SC_z(Ma,\alpha,\delta_e) \\ M &= Q(Ma,h)SlC_m(Ma,\alpha,\delta_e) \end{aligned} \tag{12}$$

Here, h is the altitude and Q, S, Ma, δ_e and l denote dynamic pressure, reference surface, Mach number, elevator deflection and reference length, respectively. The aerodynamic coefficients can be decomposed into the following structure:

$$\begin{aligned} C_Z &= C_{Z0}(Ma,\alpha) + C_{Z\delta_e}(Ma,\alpha)\delta_e \\ C_M &= C_{M0}(Ma,\alpha) + C_{Mq}(Ma,\alpha)lq/(2V) + C_{M\delta_e}(Ma,\alpha)\delta_e \end{aligned} \tag{13}$$

where V is absolute velocity and $C_{Z\delta_e}$ and $C_{M\delta_e}$ are the derivatives of the aerodynamic coefficients with respect to the elevator deflection angle. Note that in the aerodynamic data set of [9] C_z and C_M are actually not affine in δ_e. Still, this assumption is well justifiable as they are almost affine.

Using Eq. 12, Eq. 13 and $u = V\cos\alpha$, the nonlinear missile dynamics can be written as

$$\begin{bmatrix}\dot{w}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}0 & V\cos\alpha\\ 0 & \frac{QSl^2}{2VI_y}C_{Mq}(Ma,\alpha)\end{bmatrix}\begin{bmatrix}w\\ q\end{bmatrix} + \underbrace{QS\begin{bmatrix}\frac{1}{m}C_{Z0}(Ma,\alpha)\\ \frac{l}{I_y}C_{M0}(Ma,\alpha)\end{bmatrix}}_{\hat{f}(Ma,\alpha)} + \begin{bmatrix}\frac{QS}{m}C_{Z\delta_e}(Ma,\alpha)\\ \frac{QSl}{I_y}C_{M,\delta_e}(Ma,\alpha)\end{bmatrix}\delta_e \tag{14}$$

The system Eq. 14 is already almost in the form of an LPV model. The only remaining step is a function substitution of $\hat{f}(Ma,\alpha)$, such that $\hat{f}(Ma,\alpha) = A_1(Ma,\alpha)w$. For this purpose the method proposed in [2] is used. It exploits the relation $w = V\sin\alpha$ by introducing

$$C_{Zbar} = \begin{cases}0, & \text{if } \alpha = 0\\ C_{Z0}/(\sin\alpha), & \text{otherwise}\end{cases}, \quad C_{Mbar} = \begin{cases}0, & \text{if } \alpha = 0\\ C_{M0}/(\sin\alpha), & \text{otherwise}\end{cases} \tag{15}$$

With Eq. 15 the missile dynamics can be written in LPV form with the parameter vector $[Ma\ \alpha]$ as

$$\begin{bmatrix}\dot{w}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}QSC_{zbar}/mV & V cos(\alpha)\\ QSlC_{mbar}/I_yV & QSl^2C_{mq}/2I_yV\end{bmatrix}\begin{bmatrix}w\\ q\end{bmatrix} + \begin{bmatrix}QSC_{z\delta_e}/m\\ QSlC_{m\delta_e}/I_y\end{bmatrix}\delta_e \tag{16}$$

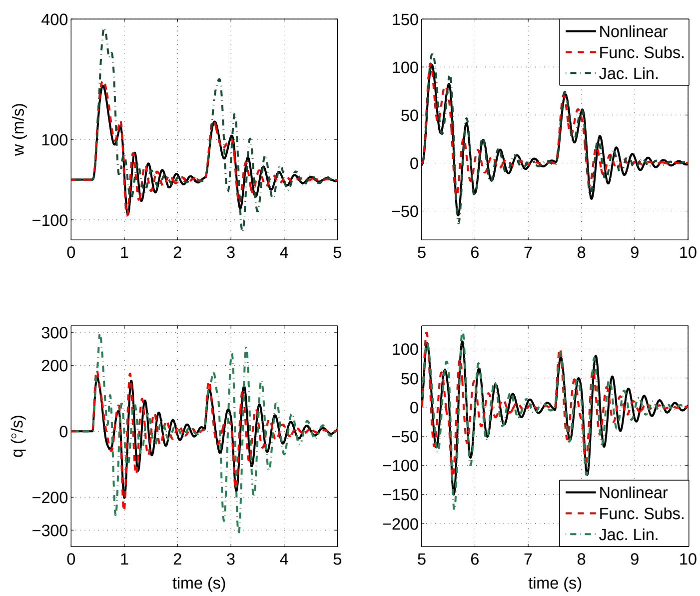


Fig. 3 Comparison of linearly scheduled Jacobian linearized and function substitution based LPV systems

A comparison between a Jacobian linearized and a function substitution based LPV system is presented in Fig. 3. In the simulation, the velocity is decreasing from Mach 2.5 to Mach 1 and the angle of attack range is between 0 and 20 degrees. The Jacobian linearized LPV system seems better at the low angle of attack regime. The worse accuracy of the function substitution based model in this region most likely stems from the affine approximation of the dependence of the aerodynamic coefficients on the elevator deflection angle. This assumption is not required in the Jacobian linearization case. Although Jacobian linearized LPV system has this advantage, when the whole flight regime including the high angle of attack regime is examined, it can be said that the function based LPV system is the better approximation. Hence, the controller design will be applied onto the function substitution based LPV system.

3.2 Controller Design

Based on the function substitution LPV model described in the previous section, an induced L_2-norm LPV controller is designed for the nonlinear missile model. The weighting scheme introduced in Section 2.2.2 is used to specify the performance requirements of the controller. The major aim of the controller is to provide good tracking performance in the acceleration (a_n). First of all, a state transformation is applied on the system, in order to turn the synthesis into a state feedback problem. The new states are the acceleration (a_n) and the pitch rate q which can both be measured by an inertial measurement unit (IMU). The controller synthesis is conducted using 64 grid points to cover the flight envelope which are:

- $\alpha = [0:3:21]^\circ$
- $Ma = [1\ \ 1.2\ \ 1.3\ \ 1.4\ \ 1.5\ \ 1.6\ \ 2\ \ 2.4]$

It shall be emphasized that the Mach number gridding needs to be finer in the transonic region, where the system dynamics are changing critically. The parameter variation rates are bounded by $\left|\dot{Ma}\right| \leq 0.2$(Ma/s) and $|\dot{\alpha}| \leq 100(^\circ/\mathrm{s})$. These values are chosen with respect to the flight envelope of the missile [10].

The structure of the parameter dependent R is chosen as:

$$R(Ma,\alpha) = R_0 + Ma_i R_1 + Ma_i^2 R_2 + \alpha_i R_3 + \alpha_i^2 R_4$$

This choice of $R(Ma,\alpha)$ is motivated by using a few simple basis functions to keep the computational burden of the synthesis low.

3.3 Optimization of the Weighting Functions

Finding suitable weighting functions is not an easy task. In order to handle this problem, the weighting functions are parameterized and optimized over all the grid

points. The roll-off frequency (w_{ro}) and the low frequency gain (k_u) of W_u are parameterized with respect to the most dominant flight parameter, namely the velocity which has a large impact on the dynamics.

$$\omega_{ro} = w_u(1) + w_u(2)Ma_i, \quad k_u = w_u(3) + w_u(4)Ma_i \tag{17}$$

The corresponding control effort penalty is depicted in Fig. 4. As seen in the Fig. 4, as the Mach number increases, the control effort is penalized more because control power is higher than at low Mach numbers, i.e. the missile needs less elevator deflection angle to achieve the same accelerations at high Mach numbers.

The gain and bandwidth of W_y is not only parameterized with respect to the Mach number but also the angle of attack. Moreover, the minimum bandwidth of the closed system is parameterized as a function of the natural frequency of the missiles short period dynamics (ω_n). With this parameterization less bandwidth is demanded at low velocities, where the open loop system is slower than at high velocities, see Fig. 4. Similarly, the tracking performance is less penalized at higher angle of attacks, because the open loop bandwidth is lower in comparison to lower angle of attacks.

$$\omega_b = \omega_n(Ma, \alpha)w_y(1), \quad k_y = w_y(2) + w_y(3)Ma_i - w_y(4)\alpha_i \tag{18}$$

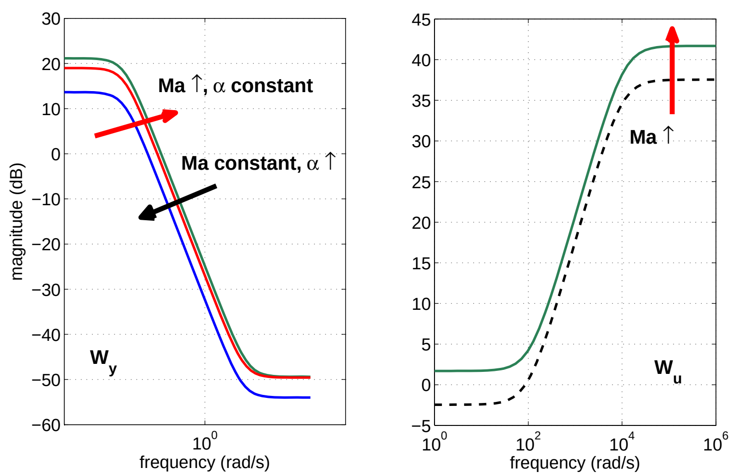


Fig. 4 Performance penalty W_y and control effort W_u

With the weighting functions W_u and W_y only a general trend can be specified for the controller objectives, as shown in Fig. 4. Detailed performance and robustness requirements are usually not given in terms of input/output gains but as a mixture of frequency and time domain criteria. For the present missile model these require-

ments are defined in Table 1. Note that the demanded settling and rise time depend on the Mach number to better exploit the capabilities of the missile. To ensure that the controller is robust, classical gain and phase margin requirement are used.

Table 1 Optimization parameters

Inequality Constraints	Value	Demands	Value
Phase Margin	$50-90°$	Settling Time	$0.5/Ma$ s
Gain Margin	10-20 dB	Rise Time	$0.2/Ma$ s
Undershoot	$\leq 20\%$		
Overshoot	$\leq 10\%$		
Gamma Value	[1 2.9]		

Using these criteria (Table 1), an optimization problem is specified. The tuners of the optimization are the free parameters in the weighting functions, see Eq. 17 and Eq. 18. The aim of the optimization is to find suitable weighting functions that minimize rise and settling time while not violating any of the specified constraints. The problem is solved for each point of the synthesis grid using the Matlab optimization environment MOPS [11]. At each grid point, it successfully surpasses the demand values and satisfies the constraints. Table 2 presents the optimized tuners, w_u and w_y, respectively.

Table 2 Optimization results of weighting tuners

W_y	Value	W_u	Value
$w_y(1)$	0.55	$w_u(1)$	90
$w_y(2)$	0.1	$w_u(2)$	9.125
$w_y(3)$	0.995	$w_u(3)$	0.4237
$w_y(4)$	0.018	$w_u(4)$	0.33

4 Nonlinear Simulations

To assess the robustness performance of the design, $\pm 10\%$ uncertainty are considered in mass, center of gravity, inertia, aerodynamic forces and moments. Nonlinear simulations are performed for every combination of minimum and maximum values of these uncertainties. This results in 512 simulations.

The simulation results are presented in Fig. 5 and Fig. 6. During the simulation, step signals in the reference acceleration are applied between 250-50 m/s^2 depending on the velocity, Fig. 5. All of the nominal results are given in bold lines. Overall, the degradation of the performance due to the considered uncertainties is very low.

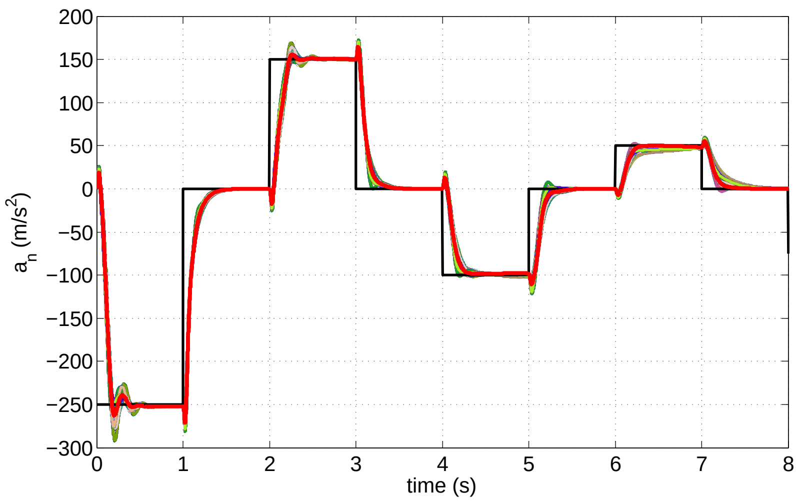
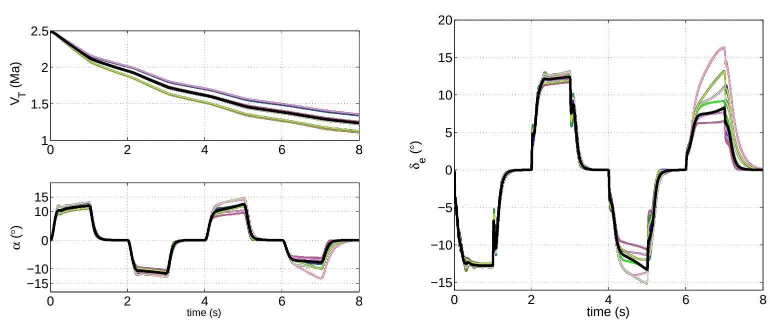


Fig. 5 Tracking performance of the LPV controller

The scheduling parameters are illustrated in Fig. 6(a). During the simulation the angle of attack is changing from 0 to 16 degrees. The velocity is decreasing from $Ma = 2.5$ to $Ma = 1$. In addition, the altitude is also changing from 2500 to 4000 meters. Note that the controller is not scheduled with respect to these altitude changes. The control surface deflections are depicted in Fig. 6(b). As seen in the figure, the



(a) Scheduling parameters (b) Elevator deflection

Fig. 6 Time histories of the nonlinear simulations

deflection angles are between ± 18 degrees which is within the limits of the considered actuator system. In the transonic region, the control effort increases drastically for the uncertain cases in comparison to the nominal simulation. In the worst case 15 degrees of deflection have to be applied instead of 5 degrees to achieve the same

performance. However, 15 degrees are still within the limits of the actuator system. The difference in the rest of the flight is not very noticeable.

5 Discussion and Conclusion

An LPV controller synthesis for a tail controlled missile is presented in this paper. First, the nonlinear missile dynamics are brought into an LPV form using the innovative method of function substitution, see [2]. It is shown that the resulting LPV system captures the behavior of the nonlinear missile over wide operation conditions better than a classical LPV modeling approach based Jacobian linearization. Second, a state feedback LPV controller in the induced L_2-norm framework (see [1]) is designed for the LPV model. A multiobjective optimization problem is used to tune the weighting functions for the controller design. Robust performance simulations are conducted with $\pm 10\%$ uncertainty on all relevant flight parameters. The results of these nonlinear simulations show that the proposed controller synthesis method is capable of providing the required level of performance for the missile model. In the future, the complete flight envelope shall be covered by the LPV controller including the very challenging boost phase.

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