

Flight Control Algorithms for a Vertical Launch Air Defense Missile

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Abstract The necessity of high maneuverability and vertical launching require thrust vector control additional to aerodynamic control. That hybrid usage of aerodynamic and thrust vectoring controls effectively increases the agility of the missile against air defense threats. This requirement and the rapidly changing dynamics of this type of missiles renders the guidance and control design critical. However, the findings suggest that classical guidance and control design approaches are still valuable to apply and can have successful performance within the effective flight envelope. It is very rare that a study concerns from detailed dynamics and analysis of the dynamics covering flight mission and algorithms. In this study, together with the modeling of the agile dynamics of a vertical launch surface to air missile and the corresponding thrust forces and moments depending on linear supersonic theory, the application of the flight control algorithms are presented. Two classic linear autopilot structures are studied. During autopilot design process, an additional term related to short period dynamics of boost phase is proposed and the drastic effect of this term is shown. In addition to control algorithms, guidance algorithms are also defined to fulfill the mission of the missile. Body pursuit algorithm is applied for rapid turnover maneuver and midcourse guidance. Proportional navigation guidance is chosen for terminal phase. In addition, an alternative maneuvering technique is proposed to reduce further side slip angle during vertical flight.

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1 Introduction

This paper presents the practical application of guidance and control methods for a vertical launch surface to air missile (VLSAM). Through the paper, the challenging dynamics of the mentioned missile is briefly presented with the modeling of the thrust vector control forces and moments, that are formed by the jet vane deflections, originating from the linear supersonic theory. There are many advanced control methods for such a rapidly varying dynamics. However, the authors of this paper advocate from a practical point of view that developing advanced control techniques should be an option only after classical control techniques have been proven to be inadequate. Hence, classical control techniques and ad-hoc gain scheduling is applied for the VLSAM. The autopilot design is pursued separately for the mid-course and terminal guidance phases of the flight. Angle autopilots are designed for the mid-course, including the rapid turnover maneuver and acceleration autopilot for terminal guidance phase respectively. The gains of the autopilot are scheduled with respect to time during boost phase and Mach for the post boost phase. The performance of the autopilots are analyzed within nonlinear simulation.

The effect of the axial acceleration during the boost phase is emphasized. Recent studies covering boost phase do not present such a term. The effect is illustrated within the simulation results regarding the comparison of schedules linearized system with the nonlinear system. The second important issue is to define a constant hybrid control ratio that interconnects the thrust vector with aerodynamic control; a singular value analysis of the linear control influence is conducted for that reason.

In addition, autopilots are integrated with body pursuit and proportional navigation guidance (PNG) guidance schemes. The overall guidance and control design is tested for a defense maneuver to defeat an approaching target. In literature, a study regarding the initial roll maneuver for interceptors based on fuzzy guidance has been found [1]. Here an alternative maneuver based on a basic approach: initial roll command generation to minimize side slip angle during vertical flight is studied which is very practical and efficient to implement.

2 Modeling of the Vertical Launch Missile Dynamics

Dynamic modeling of the VLSAM is carried out by implementing the well known Newton-Euler equations with rigid body assumption. The VLSAM, analyzed in this paper, is axi-symmetric and has a blunt nose. It is a tail controlled missile and uses both the aerodynamic tail fins and jet vanes. Two main coordinate systems as the body coordinate system (B) and the earth fixed inertial coordinate system (E) are defined and the equations of motion are derived with respect to them. The origin of the body axis system is assumed to be at the final center of gravity location after burnout. Also, since the propellant of the missile is burning throughout the flight, the mass, inertia and the position of center of gravity are formulized as a function of thrust and total impulse values and included in the model. Hence, since the thrust

and impulse are modeled as a function of time, the mass, inertia and center of gravity position of the missile change as a function of time. Hence, the translational and rotational motion of the missile can be written as Eq. 1. Detailed information about the dynamics and aerodynamics can be found in [2].

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} rv - qw + (F_{Ax} + F_{Gx} + F_{Tx})/m \\ pw - ru + (F_{Ay} + F_{Gy} + F_{Ty})/m \\ qu - pv + F_{Az} + (F_{Gz} + F_{Tz})/m \\ (M_{Ax} + M_{Tx})/I_{xx} \\ (M_{Ay} + M_{Ty} - pr(I_{xx} - I_{yy}))/I_{yy} \\ (M_{Az} + M_{Tz} - pq(I_{xx} - I_{yy}))/I_{yy} \end{bmatrix} \quad (1)$$

Here, $\bar{F}_{A(x,y,z)}^{(B)}$, $\bar{F}_{T(x,y,z)}^{(B)}$ and $\bar{F}_{G(x,y,z)}^{(B)}$ are the Cartesian components of the aerodynamic, thrust and gravity forces. $\bar{M}_{A(x,y,z)}^{(B)}$, $\bar{M}_{T(x,y,z)}^{(B)}$ and are the Cartesian components of the aerodynamic and thrust moments. u, v, w are the components of missile velocity in body coordinates, p, q, r are the missile body rates, m is mass. As, the VLSAM in this study is axi-symmetric and has cruciform geometry, $I_{yy} = I_{zz}$.

As for the calculation of the rates of the Euler angles, to avoid the singularity when the pitch angle is equal to $\mp 90^\circ$, direction cosine matrix (DCM) formulation is rather preferred to the Euler angle formulation to avoid the singularity problem. Although the quaternion formulation is computationally more efficient, it is not chosen because the DCM is more practical to apply and interpret physically.

2.1 Aerodynamic Forces and Moments

The aerodynamic forces/moments are functions of dynamic pressure (Q_d), missile reference area (S) and the aerodynamic force coefficients, i.e. C_i 's. Hence the aerodynamic force vector in matrix representation can be written:

$$\begin{aligned} \bar{F}_A^{(B)} &= [F_{Ax} \ F_{Ay} \ F_{Az}] = Q_d S [C_x \ C_y \ C_z] \\ \bar{M}_A^{(B)} &= Q_d S [C_l \ C_m \ C_n] + \begin{bmatrix} 0 \\ (x_{c_{ref}} - x_c(t))F_{Az} \\ (x_{c_{ref}} - x_c(t))F_{Ay} \end{bmatrix} \end{aligned} \quad (2)$$

The aerodynamic force/moment coefficients as nonlinear functions of flight variables; $Ma, \alpha, \beta, \delta_a, \delta_e, \delta_r$ where Ma is the Mach number, α is the angle of attack and β is the angle of sideslip, $\delta_a, \delta_e, \delta_r$ are the aileron, elevator, rudder deflections of aerodynamic control surfaces. $x_{c_{ref}}$ is the final, i.e. after the propellant burn-out, and $x_c(t)$ is the instantaneous position of the center of gravity, Q_d is the dynamic pressure, S is reference surface. the VLSAM has high angle of attack flight regime, so that an aerodynamic database is created including ± 90 degrees of angle of attack

and cross coupling terms. The details of the aerodynamic modeling and analysis of the VLSAM can be found in [2].

2.2 Thrust Forces and Moments

The thrust forces and moments, are generated by deflecting the thrust vector by the jet vanes located at the nozzle exit of the missile. The magnitude of thrust force (T), duration of the boost phase, the time to reach the maximum thrust level and the geometrical properties of the jet vanes are the critical parameters for thrust vectoring. These parameters are adapted to achieve desired maneuvering capabilities for the VLSAM [3]. The maximum thrust vectoring forces and moments are dependent on the maximum jet vane deflection angle and vane characteristics such as thickness (t_k), chord (c) and thrust motor characteristics as nozzle exit pressure (P) and flow velocity (M_∞). To determine the lift and drag forces, linear supersonic theory is directly applied. The area of the jet vane is not changing or may be neglected. There are studies that include the jet vane erosion phenomenon which effects the lift and drag forces created by vane deflections, [4], [5], [6] and [7].

$$L = \frac{PC_L\gamma M_\infty^2 S_{jv}}{2}, \quad D = \frac{PC_D\gamma M_\infty^2 S_{jv}}{2} \quad (3)$$

Here, lift and drag force coefficients (C_L and C_D) are functions of deflection of jet vanes (δ_{jv}), nozzle exit pressure, thickness and chord of the jet vanes:

$$C_L = \frac{4\delta_{jv}}{\sqrt{M_\infty^2 - 1}}, \quad C_D = \frac{4}{\sqrt{M_\infty^2 - 1}} \left(\delta_{jv}^2 + \left(\frac{t_k}{c}\right)^2 \right) \quad (4)$$

Within the scope of this paper, the area of the jet vanes are assumed to be fixed. Linear supersonic theory may sometimes overestimate the lift and drag forces, and it has to be verified with 3D computational fluid dynamics analysis and experiments. However, it is known that the accuracy of linearized theory is high when jet vanes are located at enough distance to with respect to each other and outside the nozzle [8]. Also, the dynamics between the jet vane deflection and total thrust deflection is taken as unity, because of the high the inflow rate [9]. The moments created by the deflection of the jet vanes are calculated by using the forces and the moment arms which are the distance between the nozzle exit diameter and jet vane center of pressure and the distance between the jet vane center of gravity and missile center of gravity.

3 Autopilot Model

There are studies on different algorithms for the control techniques applied at high angle of attack flight regimes. They can be mentioned as the robust control design for IRIS-T [10], the sliding mode controller [11], the adaptive control [12] and some nonlinear control strategies comparison with classical control [13]. As mentioned earlier, the aim of this study is to investigate the applicability of the classical control design techniques on that challenging VLSAM dynamics and identify the possible advantages and disadvantages. Other control techniques may then come into picture to defeat the discrepancies that the classical control design techniques cannot handle. In what follows, it will be demonstrated that autopilots designed with classical control techniques can meet the mission requirements of the VLSAM.

3.1 Linearized Systems

The state space matrix calculated with the Jacobian linearization of the nonlinear missile dynamics, at different instants of the vertical flight, can be written in the following form to express the control efficiency originating from the control surfaces separately:

$$\Delta \dot{\bar{x}} = \hat{A} \Delta \bar{x} + \hat{B}_A \bar{u}_A + \hat{B}_T \bar{u}_T \quad (5)$$

where $\bar{u}_A = [\delta_{Aa} \ \delta_{Ae} \ \delta_{Ar}]$ and $\bar{u}_T = [\delta_{Ta} \ \delta_{Te} \ \delta_{Tr}]$, and \hat{B}_A, \hat{B}_T are the control matrices for the aerodynamic and thrust vector control. Here, $\Delta \bar{x} = [\Delta U \ \alpha \ \beta \ p \ q \ r \ \phi \ \Delta \theta]$ is in general form. The eigenvalues and system matrices of vertical flight at different velocities and altitudes before and after burnout are given in [14].

To apply linear control techniques, nonlinear missile dynamics is decoupled into three simplified representations of the overall motion since there is no clear distinction between lateral and longitudinal dynamics because of its axi-symmetry property. The pitch plane state space equations can be formulated as:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha - \dot{u}/U & 1 + Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{A\delta_e} & Z_{T\delta_e} \\ M_{A\delta_e} & M_{T\delta_e} \end{bmatrix} \begin{bmatrix} \delta_{Ae} \\ \delta_{Te} \end{bmatrix} \quad (6)$$

where $M_\alpha = \frac{Q_d S d}{I_{yy}} C_{m\alpha}$, $Z_\alpha = \frac{Q_d S}{mU} C_{z\alpha}$, $M_q = \frac{Q_d S d^2}{2U I_{yy}} C_{mq}$, $Z_q = \frac{Q_d S d}{2U^2 m} C_{zq}$, $M_{A\delta_e} = \frac{Q_d S d}{I_{yy}} C_{mA\delta_e}$, $Z_{A\delta_e} = \frac{Q_d S}{mU} C_{zA\delta_e}$, $M_{T\delta_e} = \frac{T_z l_x}{I_{yy}}$, and $Z_{T\delta_e} = \frac{T_z}{mU}$ including aerodynamic derivatives e.g. $C_{m\alpha}$, $C_{z\alpha}$ etc.

3.1.1 Enhanced Short Period Approximation

The addition of the term \dot{u}/U as seen in Eq. 6 enhances short period approximation [15]. Conventional short period approximation assumes that the directional velocity component of the air vehicle is constant ($\dot{u} = 0$), however the missile under study,

especially accelerates rapidly in the boost phase. Thus, the \dot{u}/U becomes significant especially at the beginning of the vertical climb.

In Fig. 1, the normal acceleration time histories for a given longitudinal control input is presented to show the comparison between linearized and nonlinear models. Here, the linearized systems are scheduled with respect to time in the boost phase. As it is seen from the figure, the linear system behavior is drastically separated from the nonlinear system behaviour especially where \dot{u}/U is high, i.e. at the beginning of the boost phase. Thus, as for the acceleration autopilot design, it is crucial to add \dot{u}/U compensation to the short period approximation when the missile velocity is comparably low. Otherwise, the normal acceleration controller, designed without \dot{u}/U compensation, may show either inadequate performance or unstable behavior [15].

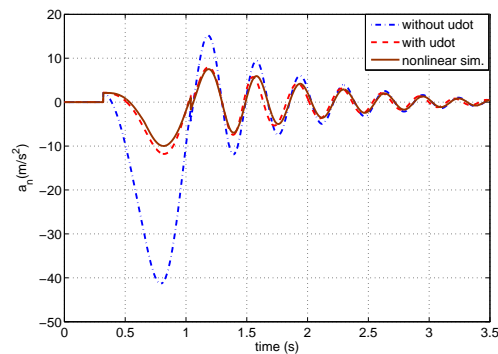


Fig. 1 Effect of missile axial acceleration in linearized missile acceleration dynamics

3.2 Thrust Vector-Aerodynamic Control Effectiveness

Control surfaces of such missiles are generally actuated using the same servo actuator. The challenging design problem is to set the "hybrid control ratio" which is directly related with the desired control capability. This ratio has to be considered together with the mission requirements and the control effectiveness. A static control effectiveness analysis is conducted for that purpose. As expected, at low velocities aero control is less effective than TVC and becomes powerful as the speed increases. However, it loses control efficiency at high angle of attack values and also at relatively high altitudes. The control effectiveness analysis of the aerodynamic and thrust vectoring controls for the VLSAM at different altitudes is shown in Fig. 2. Here, the singular values of the \hat{B} matrices are computed. TVC efficiency stays nearly constant. This is an expected result that only the total mass of the missile is decreasing while thrust level is nearly constant and it does not effect the TVC ef-

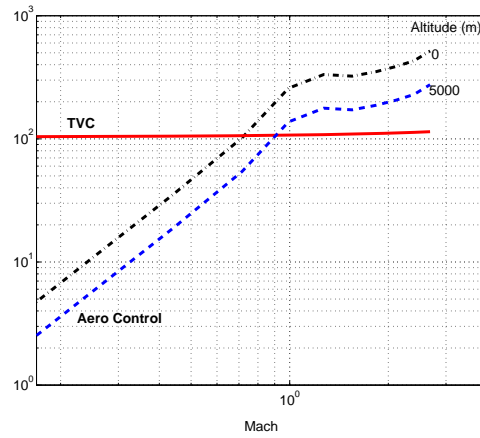


Fig. 2 Maximum singular value of B matrix

efficiency much. Further, the aerodynamic control becomes more effective than TVC after 0.7 or 0.9 Mach depending on the altitude. In order to have maximum maneuverability, the control allocation between the two control schemes is distributed as 1, but a detailed analysis and optimization study is done for this purpose [14].

3.3 Autopilot Simulations

In order to be used for the mid-course guidance, autopilot is designed to operate on roll and pitch angles [16]. In that phase, autopilot gains are time scheduled. The performance of autopilot for pitch/roll angle reference commands and deflection histories of aileron and elevator deflections are presented in Fig. 3(a) and Fig. 3(b).

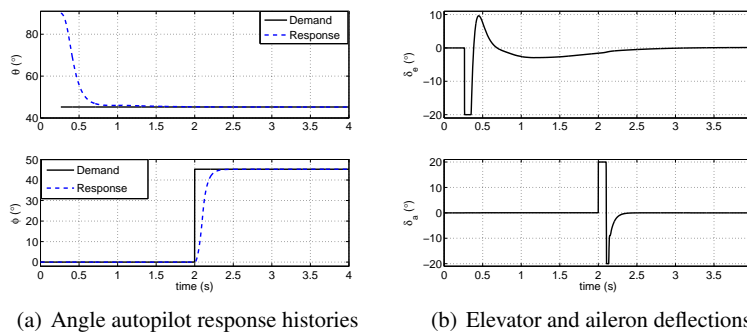


Fig. 3 Angle autopilot nonlinear simulations

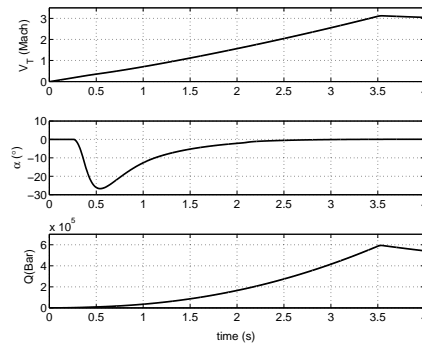


Fig. 4 Velocity, angle of attack and dynamic pressure histories

Time histories of critical parameters are also depicted in Fig. 4. Although large angle difference (45°) are demanded for both the roll and pitch attitudes, autopilot results are quite successful considering high variation of parameters such as α ($0 - 30^\circ$), speed (0-3 Ma) and drastic increase in dynamic pressure.

For the terminal guidance, acceleration autopilot is designed using the 3-loop acceleration autopilot scheme [13]. Autopilot gains are scheduled with respect to Mach. The performance of the autopilot to square command input of 15g is illustrated in Fig. 5(a) and the elevator deflection in Fig. 5(b). Since the speed is decreasing and demand stays constant in magnitude (15g), control commands increase to compensate this kinetic energy loss.

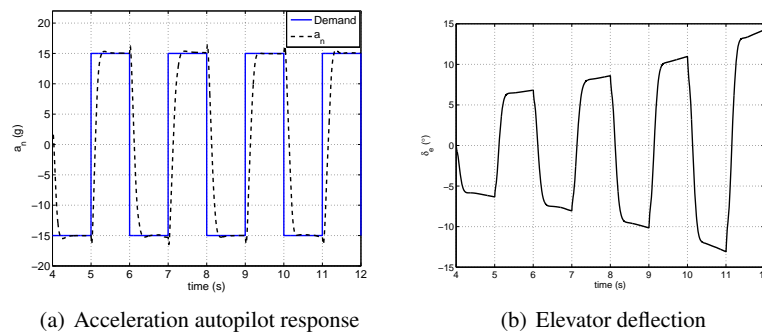


Fig. 5 Tracking performance of acceleration autopilot

4 Guidance Algorithms

One of the main advantages of vertical launching is the engagement capability to the targets in all possible directions that brings the necessity to direct the missile to the plane of motion of target as soon as possible. Here, body pursuit guidance is a possible candidate, and simple to apply, to align the missile body axis to the line of sight. This procedure tries to minimize the look angle and effectively increases the target detection possibility. The aim of the body pursuit guidance algorithm is to produce reference body angle commands to be processed in the previously designed angle command controller. As for the terminal guidance after the boost phase, the conventional PNG guidance methodology is chosen for its proven performance and ease of application.

In this study, the design of the guidance algorithms are divided into two phases as the mid-course and the terminal guidance. The mid-course guidance starts in the launch phase and operates until the hand-over to the terminal guidance phase. The switching condition from mid-course to terminal guidance generally depends on the current state of the missile and the target, trajectory constraints and the target detection sensor, i.e. seeker, properties. A simple switching condition is defined and set to occur when the lock-on range is less than 5 km and the field of view is less than 3 degrees. Intercept condition is defined based on the achievable minimum value of the closing distance as 1 m.

4.1 Guidance and Control Simulations

The proposed flight control algorithms are implemented in a defense scenario on a closing target that has a velocity of 1 Mach and starts a pull-up maneuver with 7g when the target to missile range becomes less than 3 km. The initial position of the target with respect to the missile is $p_T = [10, 2, -2]$ km. In Fig. 6(a), the acceleration

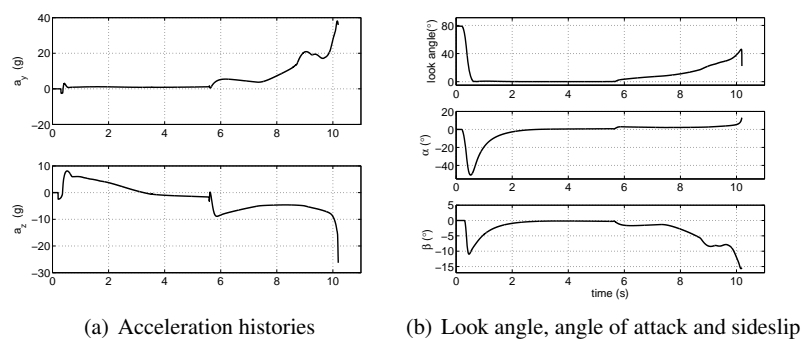


Fig. 6 Engagement simulations

time histories are presented. Fig. 6(b) illustrates time histories of the look angle, angle of attack, sideslip angle of the missile. Look angle is decreasing from 80° to 0 during the mid-course guidance phase. Whenever the guidance algorithm switches to PNG, look angle starts increasing again (the behavior of the look angle at the end is not a numerical but dynamical trend). More maneuvers on different type of targets are simulated, see [14]. In all of the target types, flight control algorithms are successful. Besides, the flight envelope has to be clarified and the overall success of the flight control algorithms has to be examined.

5 Turnover Strategy

In the previous section, the turnover strategy used in mid-course guidance was skid-to-turn strategy in which missile does not roll and yaw/pitch channel commands are applied together to maneuver the missile towards the desired direction. Here, the proposed turn-over strategy is actually a mixed ascend that turnover maneuver composed of an initial roll followed by skid-to-turn (Fig. 7). That kind of maneuvering is also used in the mid-course guidance of air to air missiles [17] and [18]. [1] discusses a turn over strategy implemented with back to turn and roll maneuvers, however a direct comparison of skid-to-turn and skid-to-turn with initial roll maneuver, and their advantages and disadvantages are not explicitly conducted.

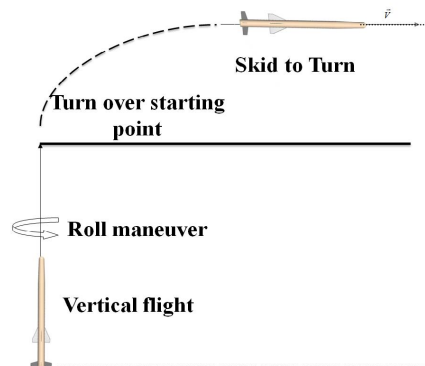


Fig. 7 Turnover with initial roll maneuver

In this turn-over strategy, the missile has an initial roll maneuver and then starts turning towards the target. The objective of the initial roll maneuver is to align the pitch plane of motion of the body of the missile to the same vertical plane with the target. Thus, after the initial roll maneuver that aligns the missile's pitch plane of motion, a maneuver in that single plane is required to head on towards the target. This brings the advantage of flying with less control effort and reduces the side slip angle values. In order to analyze the differences between standard turn-over and the

turnover with initial roll maneuver, they are implemented for different simulations for the same static target which is at $p_T = [1, 1, -0.2]$ km with respect to the missile. This necessitates a roll angle command of 45 degrees. Upon executing the roll maneuver, the engagement will become planar; so, only pitching control can be used to capture the target. Fig. 8 shows angle of attack and side slip angle time

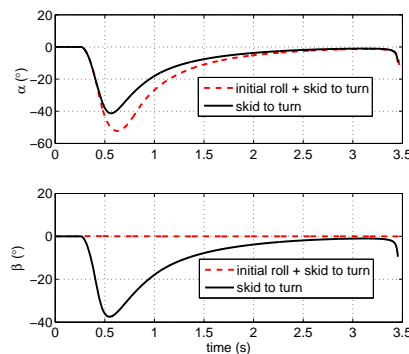


Fig. 8 Angle of attack and sideslip histories

histories compared for the standard skid-to-turn and skid-to-turn with initial roll maneuvers. With the proposed maneuver, yaw maneuver is not required to head on the target. However, as a draw back, it necessitates higher angle of attack than the standard skid-to-turn maneuver as also seen in Fig. 8.

6 Discussion and Conclusion

The paper illustrates practical flight control algorithms for an advanced missile, the VLSAM. First, six degrees of freedom flight dynamics is briefly introduced. An enhancing linearization term for the short period approximation which has not directly mentioned in literature is proposed. Its drastic effect to similarity of scheduled linearized systems and nonlinear dynamics is shown. Two different autopilots, angle and acceleration, are designed using classical control techniques for the flight envelope of the VLSAM in co-operation with guidance algorithms requirements. Aerodynamic and thrust vector control capabilities are blended in 1 : 1 ratio to have more agility. The scheduled autopilots demonstrate satisfying performance in a highly nonlinear, rapidly parameter and time varying environment which is a promising start up for design process of industrial applications.

There are also advanced guidance techniques for agile missiles, but the analysis and numerical results show that a body pursuit guidance for midcourse guidance phase and proportional guidance for terminal guidance can be directly applied to the VLSAM. Moreover, a turnover maneuver, which is the initial roll maneuver, is

also accomplished for midcourse guidance phase that decreases maneuver requirements in lateral direction. To sum up, classical approaches are still applicable for such an agile system. In order to start up a design, from industrial point of view, the flight control algorithms which are applied in this paper can be implemented easily and effectively. For further studies, maneuvers and autopilots are going to be optimized in order to maximize total energy and increase flight time. Regarding the optimization results, advanced flight control algorithms may be considered.

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