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Longitudinal control law for modern long range civil aircrafts

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Abstract The challenge in the design of commercial aircraft is the multi-disciplinary optimization over the large flight domain. This can lead to very non-linear aerodynamic and handling qualities which are influenced by a great number of parameters, such as Mach number, surfaces deflections and wing flexibility.

The presented longitudinal control laws concept copes with these difficulties by combining an adaptive controller based on a reference model and a set of dynamic feed-forwards to shape the aircraft behaviour in every condition and for every piloting task.

On top of providing a robust control of the aircraft, ensuring safety and easy to fly, the concept eases the development of flight control laws and reduces flight test effort.

1- Aerodynamic non-linearities

Aerodynamic non-linearities are mainly caused by two types of phenomena which can be combined: separations and transonic flow. They are inherent of transonic aircraft optimisation but likely to be more present with high cruise Mach number, highly tapered wings or wings with high degree of aero-structure optimization. Aerodynamic non-linearities are also very common in high-lift configurations with complex flow features, such as detached flow, wakes and flow convergence.

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Aircraft control is particularly affected by destabilisation of the pitch equilibrium (pitch-up and pitch-down effects), not controllable by a linear law.
Another challenge is that non-linearities are caused by local flow features, which are influenced by nearly all aerodynamic parameters: angle of attack, Mach number but also surfaces deflection, wing flexibility and load factor. This point also makes another approach necessary.

2- Interest for a new approach

2-1- *Classical approach*

For former civil aircrafts, whose sizing criteria were not so complex as today, the aerodynamic non linearities were weak in the normal flight domain. Then control laws were based on a classical LTI approach.
In a second time, a multimodal interpolation between LTI controllers has been introduced, to ensure a nominal aircraft behaviour close to the flight domain limits. The performance of this set of controllers depends on the non linearities, and is sufficient as long as the guaranteed performances of the aircraft are easily reached, and the safety is not jeopardized.
For a highly non-linear aircraft, it could be necessary to better control these phenomena.

2-2- *The proposed approach*

The system to control is non-linear Time Varying so a linear interpolation between controllers is not sufficient to obtain a good performance.
However, for a civil aircraft, whatever the non-linearities, the dynamic behaviour objectives are very well defined by a linear model: to size the structural components, or to cope with the pilot's sensations, a very efficient method is to guarantee that the aircraft behaviour is the one of a perfectly linear model.

This is the main reason why the proposed approach is to ensure, with a non linear control law, that the aircraft response is the one of a reference linear model.

3- Reference model-based adaptive controller

In the concept, Automatic Pilot and Human pilot give orders to the same control law. The design choice is to build an adaptive inner control loop that ensures a predictable and very stable aircraft. Then a set of feed-forwards shapes the response dynamics to a pilot order and allows to cope with manual and automatic performance specifications.

This control law is made up of 2 elements: a reference control law in addition to an adaptive control law designed to improve to reference law performance.

3-1 - ***Reference model controller***

Considering that the reference aircraft doesn't present any non-linearities in lift and pitching moment, the equations are the classical linearized ones.

(i) $$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} p_\alpha & 1 \\ m_\alpha & m_q \end{bmatrix} . \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ m_{\delta q} \end{bmatrix} . \delta_q \qquad \omega_{OI}^2 = m_q . p_\alpha - m_\alpha$$

With coefficients: $$m_\alpha = \frac{SLP_{dyn}}{I_{yy}} . Cm_\alpha \qquad m_q = \frac{SLP_{dyn}}{I_{yy}} . Cm_q$$

$$m_{\delta q} = \frac{SLP_{dyn}}{I_{yy}} . Cm_{\delta q} \qquad p_\alpha = \frac{SP_{dyn}}{mV} . Cz_\alpha$$

where S is the planform area, L is the reference chord, P_{dyn} is the dynamic pressure and I_{yy} is the aircraft inertia.

The four coefficients Cm_{α}, Cm_{q}, $Cm_{\delta q}$ and Cz_{α} are tabulated in the flight domain. The tables are easy to establish, assuming the linearity of the model.

The Airbus strategy is to control the vertical load factor Nz at the centre of gravity (CG) position by calculating an elevator deflection order δq_c^{C*}. A modal approach is used to place the short-period poles; the preferred feedbacks are the vertical load factor *Nz* and the pitch rate *q* measurements. An integral feedback ensures the unitary static gain between the commanded *Nz* (noted *Nzc*) and the measured *Nz*. Some filters on feedbacks are designed to cope with aeroelastics mode constraints (no fluttering coupling, improvement of comfort).
The elevators are represented by a third-order transfer function.
This leads to a 24th order global design plant.

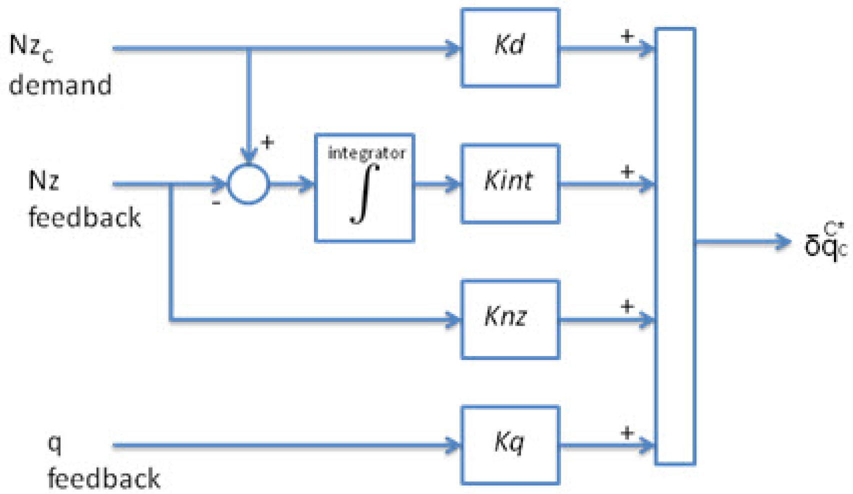


Figure 1 - Longitudinal reference control law (C*)

A judicious representation of the global filtering chain by a reduced filter allows to obtain a cascade of equations (ii) that lead to the control law feedback, without any tabulation along the flight domain. Note that a Dynamic Inversion-based design could offer the same flexibility, however the filtering chain (delays, actuators, etc) is directly taken into account in our approach.

(ii)

$$\omega_{objective} = f_\omega(m_q, p_\alpha, m_\alpha, cg, filter)$$
$$Kq = f_q \circ f_\omega(m_q, p_\alpha, m_\alpha, cg, filter)$$
$$Knz = f_{nz} \circ f_q \circ f_\omega(m_q, p_\alpha, m_\alpha, cg, filter)$$
$$Kint = f_i \circ f_\omega(m_q, p_\alpha, m_\alpha, cg, filter)$$
$$Kd = f_d \circ f_i \circ f_\omega(m_q, p_\alpha, m_\alpha, cg, filter)$$

These feedbacks are then perfectly adapted to the real aircraft, assuming that its aerodynamic coefficients are the ones above.

3-2 – ***Adaptive feedback***

Using the elevators only, it is not possible to counter fast enough the lift non-linearities; however, good handling qualities can still be obtained using an adaptive loop to control the pitching momentum.
Consider the following reference model (iii), which corresponds to the natural aircraft model outside the nonlinear effects in the flight domain:

(iii)

$$\dot{q} = m_\alpha(\alpha - \alpha_{eq}) + m_q q + m_{\delta q}(\delta q - \delta q_{eq})$$

around the equilibrium α_{eq} , δq_{eq}.
According to the real nonlinear aircraft model, any nonlinear pitching momentum Δm is therefore equal to:

(iv)

$$\Delta m = \dot{q} - m_\alpha(\alpha - \alpha_{eq}) - m_q q - m_{\delta q}(\delta q - \delta q_{eq})$$

Let $\Delta\widehat{m}(\Delta\alpha, q, \Delta\delta q)$ be the online estimate of the undesired pitching momentum using (iv). Then the adaptive elevator order δq_c^{ANL} would offset every pitching moment bias:

(v)

$$\delta q_c^{ANL} = \frac{-\Delta\hat{m}(\Delta\alpha, q, \Delta\delta q)}{m_{\delta q}}$$

The angle of attack variation from the equilibrium is deducted from (i), using only the pitch rate q:

$$\Delta\alpha = \alpha - \alpha_{eq} = \frac{q}{s - p_\alpha}$$

The elevator deflection variation from the equilibrium is modelized to capture the frequency range of the non-linearities:

$$\Delta\delta q = \delta q - \delta q_{eq} = \frac{Ts}{Ts + 1}\delta q$$

With this structure, no information about the non-linearity has to be tabulated. The control law adapts the order in real time, and the robustness to the non-linearities modelization is very good (see Fig. 2).

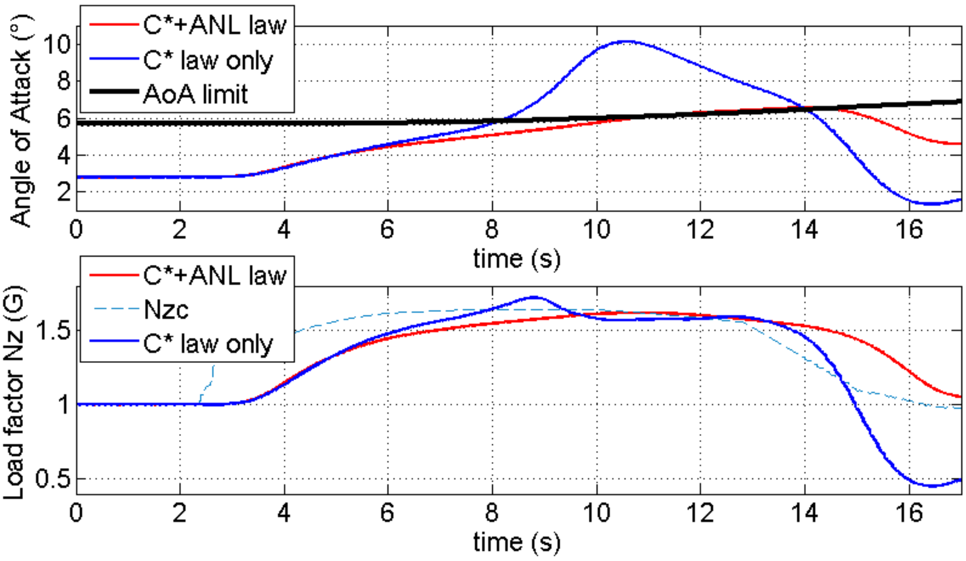


Figure 2 - Performance of the longitudinal law with and without adaptation

The robustness is less important concerning three parameters: the measurement of δq, the dephasing between the measurement of q and δq, and the dephasing between the aircraft parameters and the used informations.

The real device has to cope with many constraints (see Fig. 3):

- *The measures* $q_{mes}, \delta q_{mes}$ of $q, \delta q$ are delayed by variable and not equal time delays $TD_q(s)$ and $TD_{\delta q}(s)$. The measure q_{mes} is then filtered by $F_q(s)$ for flutter constraints. This filter dephases the information at concerned frequencies.
- The derivative operation of q_{mes} (to provide $\dot{q}$ to compute $\Delta\widehat{m}$) must cope with flutter constraints, which are strong on the pitch rate derivative. Thus the used filter *deriv(s)* cannot be simple, and it dephases hardly the derivative information at concerned frequencies. We note :

$$\dot{\tilde{q}} = deriv(s).\, q_{mes} = s \cdot F_{\frac{dq}{dt}}(s)\, q_{mes}.$$

- As all data have to be identically phased, it is necessary to add some specific delays $\varphi_q(s)$ and $\varphi_{\delta q}(s)$ on q_{mes} and δq_{mes}.
- Finally, the global delay $TD(s)$ of the law is the one represented by the addition of the acquisition time of q_{mes}, and the equivalent delay of the phase of *deriv(s)* (or equivalently the acquisition time of δq_{mes} plus the delay of its phasing filter):

$$TD(s) = delay\,\{TD_q(s)\ \cdot deriv(s)\}$$
$$= delay\,\{\varphi_{\delta q}(s) \cdot\ TD_{\delta q}(s)\}$$

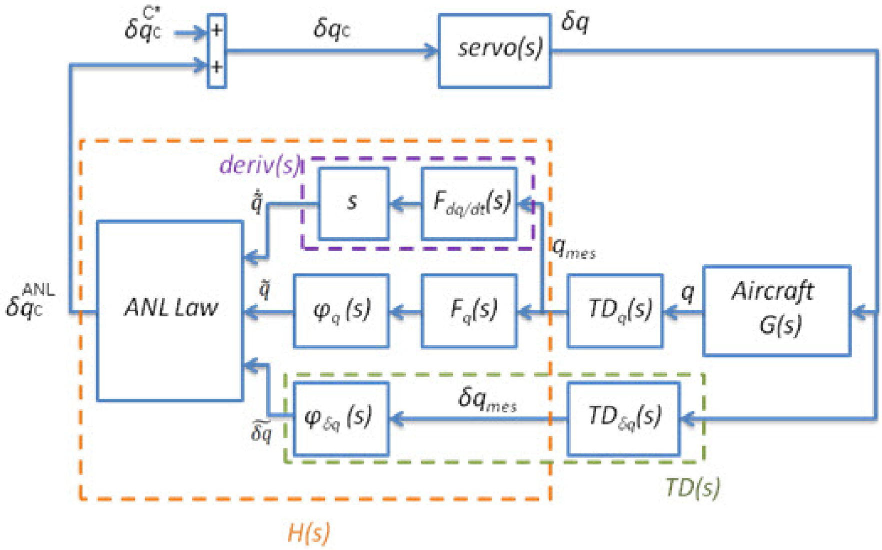


Figure 3 - ANL Law closed-loop

3-3 - ***Robustness analysis***

Representing time delays by some Pade filters, it is possible to make a Nyquist analysis of the system.

Let *servo(s)* be the elevator actuator transfer function, *G(s)* be the real aircraft transfer function between δq and q:

$$G(s) = \frac{q}{\delta q} = \frac{s - \overline{p_\alpha}}{s^2 - (\overline{p_\alpha} + \overline{m_q})s + \overline{p_\alpha m_q} - \overline{m_\alpha}}\overline{m_{\delta q}}$$

and *H(s)* be the transfer function of the device between $q_{mes}, \delta q_{mes}$ and δq_c^{ANL} (see Fig. 3 for notations), using the tabulated short-period coefficients for the reference model:

$$\delta q_c^{ANL} = \frac{-1}{m_{\delta q}}\left(\dot{\tilde{q}} - \frac{m_\alpha}{s - p_\alpha}\tilde{q} - m_q\tilde{q} - m_{\delta q}\widetilde{\delta q}\right)$$

The following transfer function is considered for the Nyquist analysis (with respect to the critical point +1):

(vi)

$$H_{(s)}^{crit} = \frac{\delta q_c^{ANL}}{\delta q_c}$$

$$= \frac{-servo(s)}{\overline{m_{\delta q}}}.\left[\left(s \cdot F_{dq/dt}(s) - \left(\frac{m_\alpha}{s - p_\alpha} + m_q\right)\varphi_q(s)F_q(s)\right)TD_q(s)\ G(s) - m_{\delta q}\varphi_{\delta q}(s)TD_{\delta q}(s)\right]$$

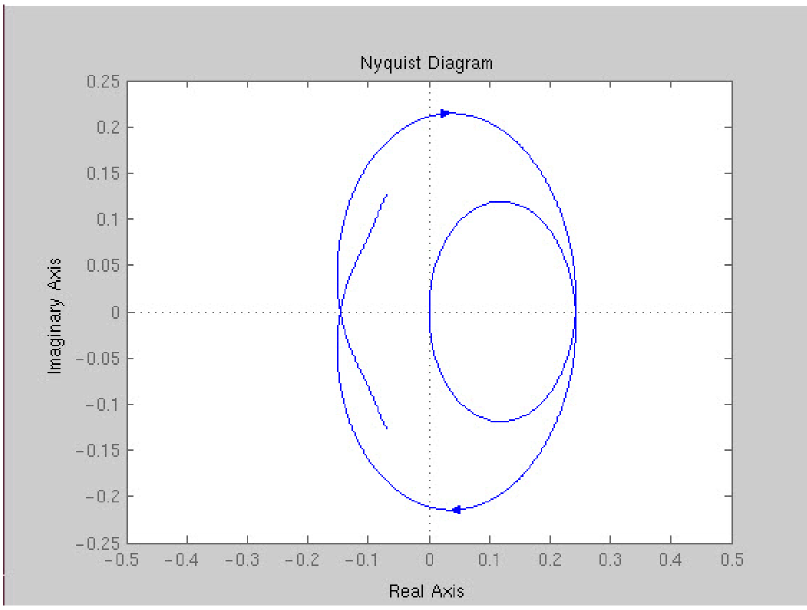
By assuming that only one parameter (or filter) is not the theoretical value, the Nyquist plot shows the stability margins with respect to this parameter (or filter). Of course, the complete exercise is done to accumulate parameters uncertainties.

3-3-1 - *Robustness to elevator information failure*

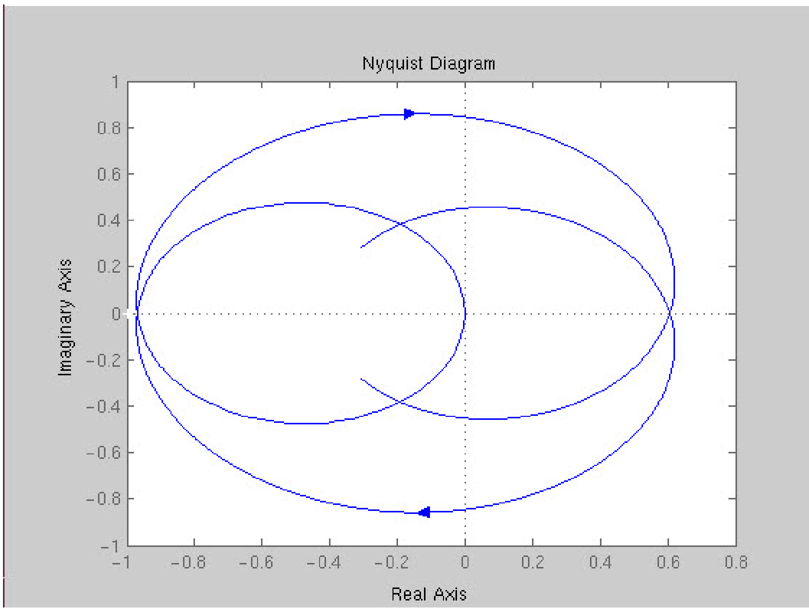
The elevators are divided into two identical servo-actuators.
The coefficient $m_{\delta q}$ has a nominal value, when the 2 elevators are working together. But this coefficient is divided by 2 when an elevator failure occurs, or strongly affected by a defect of an actuator (oscillatory failure, slow-moving device).

For this analysis, the tabulated aerodynamic coefficients m_q, m_α, p_α are considered equal to the real $\overline{m_q}, \overline{m_\alpha}, \overline{p_\alpha}$. The coefficient $m_{\delta q}$ is considered different of the real one $\overline{m_{\delta q}}$.
The impact of the non-linearity on stability is not studied here.
Only the consequences of an incorrect modelling of $m_{\delta q}$ are analyzed.
All signals are supposed well-phased so that the filters on *dq/dt* and *q* can be simplified as unit gains.
By using the reference model equation (iii) to simplify the transfer function for Nyquist-analysis (vi), we obtain:

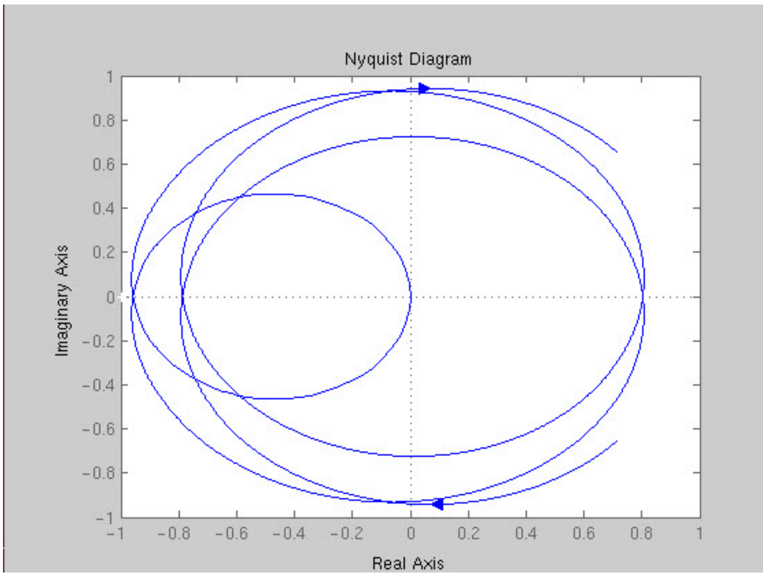
$$H_{(s)}^{crit} = \frac{-servo_{(s)}}{m_{\delta q}}.\left(\overline{m_{\delta q}} - m_{\delta q}\right).TD(s)$$



N1:Over-estimation of 25% of $\overline{m_{\delta q}}$, Δt=240ms



N2: Under-estimation of 50% of $\overline{m_{\delta q}}$, Δt=240ms



N3: Under-estimation of 50% of $\overline{m_{\delta q}}$, Δt=500ms

It appears that a strong over-estimation (ratio>>2) of $\overline{m_{\delta q}}$ could downgrade the A/C damping at its natural low frequency, but that an under-estimation (ratio 0.5) of $\overline{m_{\delta q}}$ could introduce an unstable mode whose frequency depends on $TD(s)$. $TD(s)$ represents a time delay Δt, so the "critical" frequency is $f = \frac{1}{2.\Delta t}$

It appears important to limit as much as possible the time delay Δt.

The control law is robust to an over-estimation of $m_{\delta q}$, (elevator failure) because it will slowly increase the order until the equation is balanced.
It is also robust to oscillatory failures or slow-moving devices.
Some failure detection algorithms can be coded to adapt the mδq value to the real time situation.

But the performance of the law is hardly affected if $m_{\delta q}$ is under-estimated. It is the case when the system has detected a failure of an elevator which is not failed. The gain margin of the law is not important, so this case has to be extremely improbable.
Moreover, the flexible modes are hardly impacted by an increase of control law orders, and a servo-aeroelastic coupling can occur.

To avoid such an under-estimation of $\overline{m_{\delta q}}$, the redundancy of sensors is a good mean. Some failure detection algorithms can also be implemented.
Some algorithms exist, quite simple (based on the bias between order and position), or more complex (based on Power Spectral Density analysis of the elevators motion).

3-3-2 – *Robustness to the dephasing between inputs*

Considerer now that all aerodynamic coefficients are well tabulated. The impact of an unknown delay on either q (the real delay is different than our model TD_q) or δq (the real delay is different than our model $TD_{\delta q}$) is analyzed. Of course only one has to be considered in the following equation, the other one is set to 1. Based on these assumptions, the transfer function for the Nyquist-analysis can be reduced to:

$$H_{(s)}^{crit} = -servo(s).\left(TD_q - TD_{\delta q}\right).TD(s)$$

Mathematically, it appears that the critical frequency depends on the difference D= $TD_q(s)$ - $TD_{\delta q}(s)$ and the global delay TD. For memory, the performance of the law to counter nonlinearities is robust to a difference D around 50ms. The stability of the law is jeopardized when D reaches 150ms.

Of course, the level of sensitivity linked to a dephasing depends directly on the other parameters, i.e aerodynamic coefficients errors, and the global dephasing $TD(s)$.

Then it is important to phase the q and δq chains of the control law.

3-3-3 – *Robustness to the global dephasing* $TD(s)$

Of course, if we consider that the aerodynamic coefficients are well tabulated and that phasing is dealt with, the law is absolutely stable in the linear domain, because the order is always nil.

In fact, if some parameters are not exact, or if the phasing is not perfect, an increase of $TD(s)$ will downgrade the performance to counteract a non-linearity before destabilizing theclosed-loop system in the linear domain. Nevertheless, it is clear that a strong $TD(s)$ could destabilize the linear system.
Therefore it is important for the performance of the law to minimize the global dephasing, which is linked to the necessity to avoid any aeroelastic coupling. As a consequence, the aeroelastic filters on q and $\dot{q}$ must be optimized.

4- The feed-forwards set the piloting performances.

Once the aircraft behaviour is fixed, stable and homogeneous in the flight domain, a set of dynamic feed-forwards is designed to shape the aircraft responses to piloting orders. The performance specification is not the same in every flight phase, and depends on the nature of the pilot: human or automatic pilots don't need the same A/C response to a same order.

The dynamic feed-forwards are some mathematical interpretations of the particular performance specifications of manual or automatic piloting requirements.
These feed-forwards will deal for instance with the flat lift curve characteristics during approach, in order to allow a very precise glide signal follow-up.

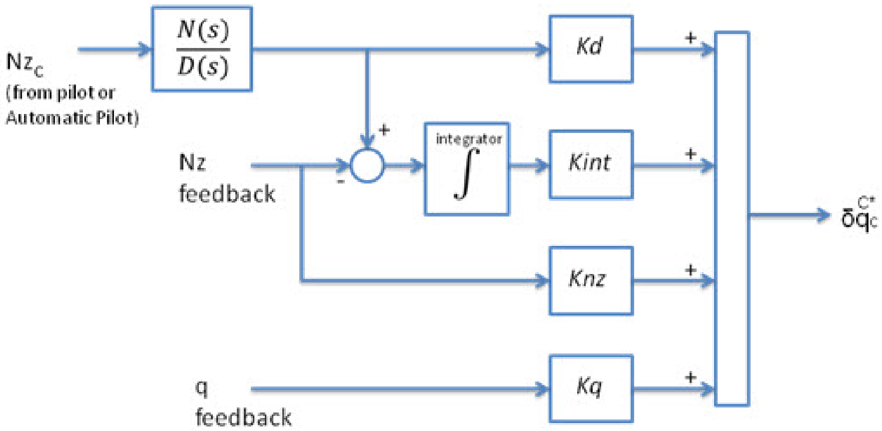


Figure 3 - Longitudinal reference control law for various piloting objectives

By modification of the feedforward, the aircraft response to a stick input will favor pitch angle stability or flight path angle stability.

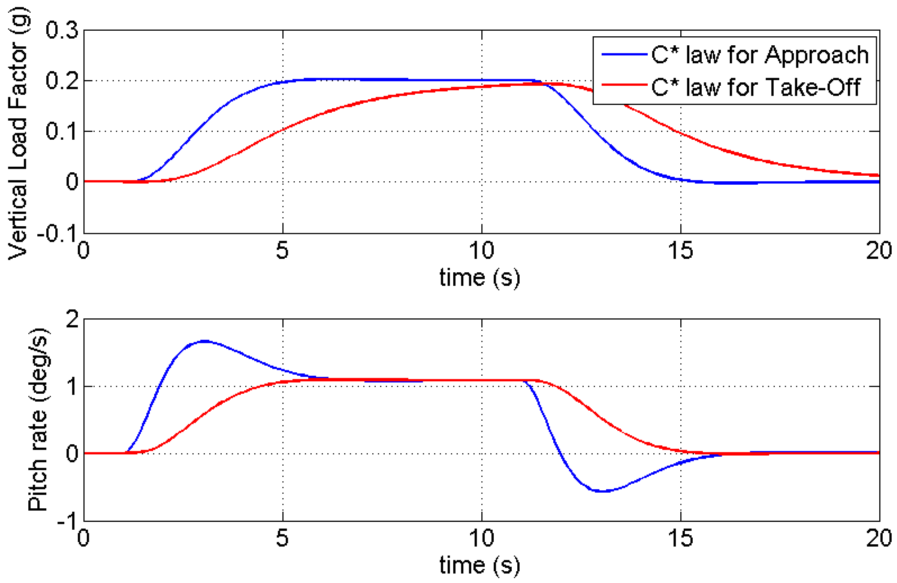


Figure 4 - Different feedforwards for different piloting objectives

5- Benefits of such a complex control law for large civil aircraft development

This control law copes with all the characteristics of the aircraft and by solving a set of equations, it provides many benefits over other approaches:

- The sizing of the aircraft during development is more robust to coefficients variations, because the control law shapes the aircraft behaviour. The requirements are easier to meet.
- The flight test campaign is shorter, because the control law is more robust to unpredicted non-linearities or other kind of inaccuracy of the models.
- The validation of the aircraft is more homogeneous in the flight domain.

This longitudinal control law approach definitively appears as a simple and robust concept to deal with the complexity of modern aircraft control.