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Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays - Part I

James P. Nelson, Dr. Mark J. Balas and Dr. Richard S. Erwin

Abstract In this paper, we develop a Direct Model Reference Adaptive Tracking Controller for mildly non-linear systems with unknown time varying input delays. This controller can also reject bounded disturbances of known waveform but unknown amplitude, e.g. steps or sinusoids. In this paper a robustness result is developed for DMRAC of mildly non-linear systems with unknown small constant or time varying input delays using the concept of un-delayed ideal trajectories. We will show that the adaptively controlled system is globally stable, but the adaptive tracking error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design. A simple example will be provided to illustrate this adaptive control method. The proof of the corollary for the application and further examples are provided in the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part II.

1 Introduction

Time delay affects many engineering, physics and biological systems [1]-[5]. These manuscripts present a firm motivation for the study of time delay systems and a brief overview of the different control approaches commonly used when delays are present. In this overview the open problem of control via the delay and constructive use of the delayed inputs is presented [5].Further,

James Nelson and Dr. Mark Balas
University of Wyoming Department of Electrical and Computer Engineering, 1000 E. Universtiy Ave. Laramie, WY 82073, e-mail: nelsonpj@uwyo.edu,mbalas@uwyo.edu

Richard S. Erwin
Air Force Research Laboratory, Kirtland AFB, NM, 87117, USA

many control systems suffer from unknown delays [6]-[7]. Often these are introduced via systems controlled through a network, e.g in [7].

In previous work [8]-[11] direct model reference adaptive control (DMRAC) and disturbance rejection with very low order adaptive gain laws for MIMO systems was accomplished. Feuntes and Balas developed an ultimate bounded-ness theorem for DMRAC in [11]. When systems are subjected to an unknown internal delay, the adaptive control theory can be modified to handle this situation [12]. However, delays appearing in the inputs or outputs of systems seem to cause more system sensitivity to the delay. A robustness result for the Direct Adaptive Control (DAC) or input delay systems was developed in [13]. A robustness result for the DMRAC of linear systems with “small” input/output delays was developed in [14] using the concept of un-delayed ideal trajectories for the development of the adaptive error system. Using the concept of un-delayed ideal trajectories and this “small-ness” assumption the results of [13] can achieved for the DMRAC of mildly non-linear systems. We will show that the adaptively controlled system is globally stable, but the adaptive error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design. A simple example will be provided to illustrate this adaptive control method. The proof of the corollary for the application and further examples are provided in the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part II.

2 Development of the Adaptive Error System Using “Undelayed Ideal Trajectories”

Our Mildly Non-Linear Plant with Unknown Delay will be modelled by the following mildly non-linear system with an input delay term and an external persistent disturbance:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t-\tau(t)) + \Gamma u_D(t) + f(x) \\ y(t) = Cx(t)\,;\; x(0) = x_0 \end{cases} \tag{1}$$

where the plant state, x(t), is an N-dimensional vector with M-dimensional control input vector, u(t), and M-dimensional sensor output vector, y(t), i.e. the plant is square. The delay $\tau(t) > 0$ is time varying and unknown. The disturbance input vector $u_D(t)$ is N_D-dimensional and will be thought to come from the following Disturbance Generator:

$$\begin{cases} u_D = \Theta\, z_D \\ \dot{z}_D = F\, z_D\,;\; z_D(0) = z_0 \end{cases} \tag{2}$$

The objective of control in this paper will be to cause the output $y(t)$ of the plant to asymptotically track the output $y_m(t)$ of an un-delayed Reference Model:

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u_m + f_m(x_m) \\ y_m = C_m x_m;\ x_m(0) = x_0^m \end{cases} \tag{3}$$

where the reference model state $x_m(t)$ is an N_m-dimensional vector with reference model output $y_m(t)$ having the same dimension as the plant output $y(t)$. In general, the plant and reference models need not have the same dimensions. The excitation of the reference model is accomplished via the vector $u_m(t)$ which is generated by:

$$\dot{u}_m = F_m u_m;\ u_m(0) = u_0^m \tag{4}$$

The reference model parameters (A_m, B_m, C_m, F_m) will be completely known. We define the output error vector:

$$e_y \equiv y - y_m \underset{t\to\infty}{\longrightarrow} 0 \tag{5}$$

and this control objective will be accomplished by an Adaptive Control Law of the form:

$$u = G_m x_m + G_u u_m + G_e e_y + G_D \varphi_D \tag{6}$$

2.1 Ideal Trajectories

We define the "Un-Delayed Ideal Trajectories" for (1) in the following way:

$$\begin{cases} x_* = S_{11}^* x_m + S_{12}^* u_m + S_{13}^* z_D \\ u_* = S_{21}^* x_m + S_{22}^* u_m + S_{23}^* z_D \end{cases} \tag{7}$$

where the ideal trajectory $x_*(t)$ is generated by the ideal control $u_*(t)$ from

$$\begin{cases} \dot{x}_* = A x_* + B u_* + \varGamma u_D + f(x_*) \\ y_* = C x_* = y_m \end{cases}. \tag{8}$$

If such ideal trajectories exist, they will be linear combinations of the reference model state and input (3) and they will produce exact output tracking in a delay-free plant (8).

By substitution of (7) into (8) using (3)-(4), we obtain the Model Matching Conditions:

$$A S_{11}^* + B S_{21}^* = S_{11}^* A_m \tag{9}$$

$$A S_{12}^* + B S_{22}^* = S_{12}^* F_m + S_{11}^* B_m \tag{10}$$

$$CS_{11}^{*} = C_m \tag{11}$$

$$CS_{12}^{*} = 0 \tag{12}$$

$$AS_{13}^{*} + BS_{23}^{*} + \Gamma\Theta = S_{13}^{*}F \tag{13}$$

$$BS_{23}^{*} = 0 \tag{14}$$

$$CS_{13}^{*} = 0 \tag{15}$$

$$f(S_{11}^{*}x_m + S_{12}^{*}u_m + S_{13}^{*}z_D) = S_{11}^{*}f_m x_m \tag{16}$$

These conditions (9)-(16) are necessary and sufficient conditions for the existence of the ideal trajectories in the form of (7).

2.2 Fixed Gain Controller

In this section only we will assume that all parameters $(A, B, C, \Gamma, \Theta, F)$ are known, as well as the solutions to the Model Matching Conditions (9)-(16). This section will help to explain the development of the adaptive scheme; it is not meant to be used in place of such a scheme. We define the state tracking error:

$$e_* \equiv x - x_* \tag{17}$$

and, from (5) and (8) , we obtain

$$e_y \equiv y - y_m = y - y_* = Cx - Cx_* = Ce_*. \tag{18}$$

Furthermore, from (1) and (8) , we have

$$\left\{ \begin{array}{c} e_* \equiv x - x_* \\ \Delta u \equiv u - u_* \\ e_y = y - y_* \\ \Delta f \equiv f(x) - f(x_*) \end{array} \right. \tag{19}$$

$$\Rightarrow \left\{ \begin{array}{c} \dot{e}_* = Ae_* + B(u(t-\tau(t)) - u_*) + \Delta f \\ \Delta u \equiv u(t-\tau(t)) - u_* = u(t-\tau(t)) - u + u - u_* \\ e_y = Ce_* \end{array} \right.$$

We define a Fixed Gain Controller:

$$u = (S_{21}^{*}x_m + S_{22}^{*}u_m + S_{23}^{*}L\varphi_D) + G_e^{*}e_y = u_* + G_e^{*}e_y. \tag{20}$$

From (19) and (20), we have

$$\begin{cases} \dot{e}_* = A_C e_* + B(u(t-\tau(t)) - u) + \Delta f \\ A_C \equiv A + BG_e^* C \\ \Delta f \equiv f(x) - f(x_*) \end{cases} . \tag{21}$$

The above can be summarized as:

If (A, B, C) is output feedback stabilize-able with a gain G_e^* and the delay equation (21) is stable, then the fixed gain controller (20) will produce local output tracking, i.e.:

$$\lim_{t\to\infty} e_y < R_* \tag{22}$$

Note that output feedback stabilization can be accomplished when

$$M + P + N_D > N \tag{23}$$

and (A, B, C) is controllable and observable; see [14]. Since (23) does not require detailed knowledge of the parameter matrices, this suggests that an adaptive control scheme might be possible.

2.3 The Adaptive Controller

The form of our adaptive controller remains (6). In this we must develop the gain adaptation laws to make asymptotic output tracking possible. We form

$$\begin{cases} \Delta G_u \equiv G_u - S_{22}^* \\ \Delta G_m \equiv G_m - S_{21}^* \\ \Delta G_e \equiv G_e - G_e^* \\ \Delta G_D \equiv G_D - S_{23}^* L \end{cases} \tag{24}$$

where the starred gains come from (9)-(16) and (20). Now, from (6), and (20),

$$\Delta u = u - u_* = \Delta G_u u_m + \Delta G_m x_m + (G_e^* + \Delta G_e) e_y + \Delta G_D \varphi_D \tag{25}$$

Then, via (18) and (25), with appropriate definitions, we have

$$\begin{aligned} \dot{e}_* &= A e_* + B(u(t-\tau(t)) - u) + B\Delta u + \Delta f \\ &= (A + BG_e^* C)\, e_* + B(u(t-\tau(t)) - u) + B \begin{bmatrix} \Delta G_u & \Delta G_m & \Delta G_e & \Delta G_D \end{bmatrix} \eta + \Delta f \\ &= A_C e_* + B(u(t-\tau(t)) - u) + B\Delta G \eta + \Delta f \end{aligned} \tag{26}$$

where,

$$\eta \equiv [u_m^T x_m^T e_y^T \varphi_D^T]^T$$

is the vector of known available signals.

We combine (18) and (26) to obtain the Tracking Error System:

$$\begin{cases} \dot{e}_* = A_C e_* + B(u(t-\tau(t)) - u) + B\Delta G\eta + \Delta f \\ e_y = Ce_* \end{cases}. \tag{27}$$

Now we specify the Adaptive Gain Laws:

$$\dot{G} = -e_y \eta^T H - aG(t) \tag{28}$$

where

$$H \equiv diag[h_{11}, h_{22}, h_{33}, h_{44}] > 0$$

is an arbitrary, diagonal, positive definite matrix. This yields

$$\begin{cases} \dot{G}_u = -e_y u_m^T h_{11} - aG_u(t) \\ \dot{G}_m = -e_y x_m^T h_{22} - aG_m(t) \\ \dot{G}_e = -e_y e_y^T h_{33} - aG_e(t) \\ \dot{G}_D = -e_y \varphi_D^T h_{44} - aG_D(t) \end{cases} \tag{29}$$

3 Robustness of the Adaptive Error System

Our closed loop Adaptive Error System becomes (27) with the above adaptive gain laws (29)

$$\begin{cases} \dot{e}_* = A_C e_* + B(u(t-\tau(t)) - u) + B\Delta G\eta + \Delta f \\ e_y = Ce_* \\ \Delta\dot{G} = \dot{G} = -e_y \eta^T H - aG(t) \end{cases}. \tag{30}$$

With the development of the above adaptive error system, recall the theorem developed in [13]

Theorem: Consider the nonlinear, coupled system of differential equations,

$$\begin{cases} \dot{e} = A_c e + f(e) + B\left(G(t) - G^*\right) z + \nu + f(x) \\ e_y = Ce \\ \dot{G}(t) = -e_y z^{\mathrm{T}} \gamma - aG(t) \end{cases}. \tag{31}$$

where G* is any constant matrix and is any positive definite constant matrix, each of appropriate dimension. Assume the following:

1. the delay-free linear part (A_c, B, C) is SPR (see [15]),
2. $\exists M_g > 0 \ni \sqrt{tr(G^* G^{*T})} \leq M_G$
3. $\exists M_v > 0 \ni \sup\limits_{t \geq 0} \|\nu(t)\| \leq M_\nu$

4. $\exists a > 0 \ni a \leq \frac{\beta}{2p_{\min}}; \beta \equiv q_{\min} - 2\mu_f p_{\max} > 0$where p_{min}, p_{max} are the minimum and maximum eigenvalues of P and q_{min} is the minimum eigenvalue of Q with respect to the Kalman-Yacubovich equations,
5. the positive definite matrix γ satisfies

$$tr(\gamma^{-1}) \leq \left(\frac{M_\nu}{aM_G}\right)^2$$

6. the nonlinear term f(x) is Lipshitz continuous at 0, i.e.

$$\|f(x)\| \leq \mu_f \|x\|$$

with

$$\mu_f < \frac{q_{\min}}{2p_{\max}}$$

Then the gain matrix, G(t), is bounded, and the state, e(t) exponentially with rate approaches the ball of radius

$$R_* \equiv \frac{\left(1 + \sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_\nu$$

We can obtain a corollary of the above theorem for the adaptive error system (30) with the following assumptions:

We will say that the unknown time varying delay $\tau(t)$ is small when

$$\begin{cases} |\tau(t)| \leq \tau_* < \infty \\ \|u(t) - u(t - \tau(t))\| \leqslant M(\tau_*) \underset{t_* \to 0}{\to} 0 \end{cases} \tag{32}$$

the above system must have output tracking to a neighborhood:

$$e_y \underset{t \to \infty}{\to} R_* \tag{33}$$

The adaptive controller will have the form:

$$\begin{cases} \dot{G}_u = -e_y u_m^T h_{11} - aG_u(t) \\ \dot{G}_m = -e_y x_m^T h_{22} - aG_m(t) \\ \dot{G}_e = -e_y e_y^T h_{33} - aG_e(t) \\ \dot{G}_D = -e_y \varphi_D^T h_{44} - aG_D(t) \end{cases}. \tag{34}$$

Using the above, we have the following corollary about the corresponding direct adaptive control strategy for the adaptive error system in 30:

Corollary: Assume the following:

1. There exists a gain, G_e^*such that the triple $(A_C \equiv A + BG_e^* C, B, C)$ is SPR (this is known to be equivalent to $CB > 0$ and the open loop transfer function

$$P(s) \equiv C(sI - A)^{-1}B \tag{35}$$

is minimum phase),
2. (32) is satisfied
3. Span(Γ) $\subseteq$ Span(B)

with a positive constants, then the output y exponentially approaches a neighborhood with radius proportional to the magnitude of the disturbance, v, for sufficiently small a and γ_i. Furthermore, each adaptive gain matrix is bounded.

This corollary provides a control law that is robust with respect to persistent disturbances and, exponentially with rate e^{-at}, produces:

$$\overline{\lim_{\tau\to\infty}} \|e(t)\| \leqslant \frac{\left(1+\sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} \|B\| \, M(\tau) \underset{t\to 0}{\to} 0.$$

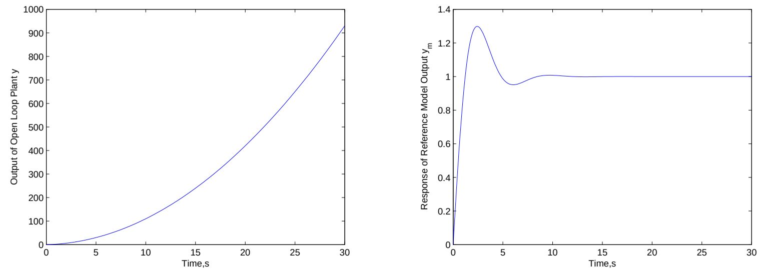
The Proof of the Corollary is provided in the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part II.

4 Simulation and Results

We will illustrate the above robust adaptive controller on the following plant:

$$\begin{cases} \dot{x} = \underbrace{\begin{bmatrix} x_2 \\ 0.3 * \sin(x_1) \end{bmatrix}}_{A(x)} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t-\tau) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\Gamma} u_D \\ y = \underbrace{\begin{bmatrix} 1 \; 0.1 \end{bmatrix}}_{C} x \end{cases} \tag{36}$$

We use step disturbances to provide simulation results for various small time varying values of delay $\tau(t)$. An adequate reference model must be developed for output tracking. The open loop output response to a step disturbance of magnitude 1 can be seen in Fig. 1(a). The desired reference model output, $y_m(t)$, for the closed loop reference model linear plant and lead controller to a step disturbance of magnitude 1 can be seen in Fig. 1(b). This reference model output was created by designing a lead controller to stabilize the plant and achieve the desired temporal response characteristics. Further simulations to illustrate this adaptive control method are provided in the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part II.



(a) Output response, $y(t)$ (b) Output response, $y_m(t)$

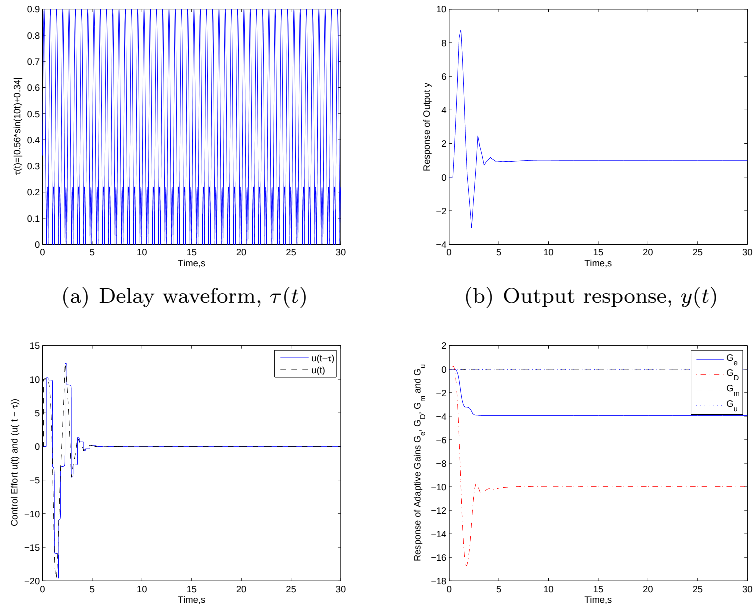
Fig. 1 Output response, (a) $y(t)$, for the open loop plant and (b) $y_m(t)$, for the closed loop reference model plant and lead controller to a step disturbance of magnitude 1.

4.1 Step Disturbances

The waveform of time varying delay $\tau(t) = |0.56 * sin(10t) + 0.34|(s)$ is shown in Fig. 2(a). The response to a step disturbance of magnitude 10 of the output response, y(t), control effort u(t), and the adaptive gains for the input delay time, $\tau(t) = |0.56 * sin(10t) + 0.34|(s)$ are shown in Fig. 2(b), 2(c) and 2(d) respectively. This simulation has shown that the adaptive controller can force a simple midly-nonlinear plant to adequately track a linear reference model. The adaptive controller can operate in the presence of “small” constant and time varying delays without any knowledge of the delay.

5 Conclusions

In this paper, we developed a Direct Model Reference Adaptive Tracking Controller for mildly non-linear systems with unknown time varying input delays. This controller can also reject bounded disturbances of known wave form but unknown amplitude, e.g. steps or sinusoids. In this paper a robustness result was developed for DMRAC of mildly non-linear systems with unknown small constant or time varying input delays using the concept of un-delayed ideal trajectories. We showed that the adaptively controlled system is globally stable, but the adaptive tracking error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design. A simple example was provided to illustrate this adaptive control method. The proof of the corollary for the application and further examples are provided in the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part II.



(a) Delay waveform, $\tau(t)$ (b) Output response, $y(t)$

(c) Control Effort, u(t) (d) Response of adaptive gains

Fig. 2 (a)Delay waveform, $\tau(t)$, (b) Output response, $y(t)$, (c) Control Effort, u(t) and (d) Response of adaptive gains for a step disturbance of magnitude 10 and $\tau(t) = |0.56 * sin(10t) + 0.34|$(s).

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