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Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays - Part II

James P. Nelson, Dr. Mark J. Balas and Dr. Richard S. Erwin

Abstract In this paper, a proof for the corollary developed for the Direct Model Reference Adaptive Tracking Control of mildly non-linear systems with unknown time varying input delays found in Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays - Part I is completed. The adaptive error system was developed for the DMRAC of mildly non-linear systems with unknown small constant or time varying input delays using the concept of un-delayed ideal trajectories. We will show that the adaptively controlled system is globally stable, but the adaptive tracking error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design. A simple example will be provided to illustrate this adaptive control method.

1 Introduction

This paper is the companion to Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays - Part I. The introduction and some of the theoretical development will be restated so it can be read as a stand alone paper. Time delay affects many engineering, physics and biological systems [1]-[5]. These manuscripts present a firm motivation for the study of time delay systems and a brief overview of the different control approaches commonly used when delays are present. In this overview the open problem of control via the delay and constructive use of the delayed

James Nelson and Dr. Mark Balas
University of Wyoming Department of Electrical and Computer Engineering, 1000 E. Universtiy Ave. Laramie, WY 82073, e-mail: nelsonpj@uwyo.edu,mbalas@uwyo.edu

Richard S. Erwin
Air Force Research Laboratory, Kirtland AFB, NM, 87117, USA

inputs is presented [5].Further, many control systems suffer from unknown delays [6]-[7]. Often these are introduced via systems controlled through a network, e.g in [7].

In previous work [8]-[11] direct model reference adaptive control (DMRAC) and disturbance rejection with very low order adaptive gain laws for MIMO systems was accomplished. Feuntes and Balas developed an ultimate bounded-ness theorem for DMRAC in [11]. When systems are subjected to an unknown internal delay, the adaptive control theory can be modified to handle this situation [12]. However, delays appearing in the inputs or outputs of systems seem to cause more system sensitivity to the delay. A robustness result for the Direct Adaptive Control (DAC) or input delay systems was developed in [13]. A robustness result for the DMRAC of linear systems with "small" input/output delays was developed in [14] using the concept of undelayed ideal trajectories for the development of the adaptive error system. Using the concept of un-delayed ideal trajectories and this "small-ness" assumption the results of [13] can achieved for the DMRAC of mildly non-linear systems. We will show that the adaptively controlled system is globally stable, but the adaptive error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood can be achieved as a result of the control design.

2 Robustness of the Adaptive Error System

In the paper: Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delay - Part I the concept of "undelayed ideal trajectories" was used to develop the adaptive error system:

$$\begin{cases} \dot{e}_* = A_C e_* + B(u(t-\tau(t)) - u) + B\Delta G\eta + \Delta f \\ e_y = C e_* \\ \Delta\dot{G} = \dot{G} = -e_y \eta^T H - aG(t) \end{cases} . \tag{1}$$

Recall the theorem developed in [13]

Theorem: Consider the nonlinear, coupled system of differential equations,

$$\begin{cases} \dot{e} = A_c e + f(e) + B\left(G(t) - G^*\right) z + \nu + f(x) \\ e_y = Ce \\ \dot{G}(t) = -e_y z^{\mathrm{T}} \gamma - aG(t) \end{cases} . \tag{2}$$

where G* is any constant matrix and is any positive definite constant matrix, each of appropriate dimension. Assume the following:

1. the delay-free linear part (A_c, B, C) is SPR (see [15]),
2. $\exists M_g\ 0 \ni \sqrt{tr(G^* G^{*T})} \leq M_G$

3. $\exists M_v\ 0 \ni \sup_{t \geq 0} \|\nu(t)\| \leq M_\nu$
4. $\exists a\ 0 \ni a \leq \frac{\beta}{2p_{\min}}; \beta \equiv q_{\min} - 2\mu_f p_{\max} > 0$where p_{min}, p_{max} are the minimum and maximum eigenvalues of P and q_{min} is the minimum eigenvalue of Q with respect to the Kalman-Yacubovich equations,
5. the positive definite matrix γ satisfies

$$tr(\gamma^{-1}) \leq \left(\frac{M_\nu}{aM_G}\right)^2$$

6. the nonlinear term f(x) is Lipshitz continuous at 0, i.e.

$$\|f(x)\| \leq \mu_f \|x\|$$

with

$$\mu_f < \frac{q_{\min}}{2p_{\max}}$$

Then the gain matrix, G(t), is bounded, and the state, e(t) exponentially with rate approaches the ball of radius

$$R_* \equiv \frac{\left(1 + \sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_\nu$$

We can obtain a corollary of the above theorem for the adaptive error system in (1) with the following assumptions:

We will say that the unknown time varying delay $\tau(t)$ is small when

$$\begin{cases} |\tau(t)| \leq \tau_* < \infty \\ \|u(t) - u(t - \tau(t))\| \leqslant M(\tau_*) \underset{t_* \to 0}{\to} 0 \end{cases} \tag{3}$$

the above system must have output tracking to a neighborhood:

$$e_y \underset{t \to \infty}{\to} R_* \tag{4}$$

The adaptive controller will have the form:

$$\begin{cases} \dot{G}_u = -e_y u_m^T h_{11} - aG_u(t) \\ \dot{G}_m = -e_y x_m^T h_{22} - aG_m(t) \\ \dot{G}_e = -e_y e_y^T h_{33} - aG_e(t) \\ \dot{G}_D = -e_y \varphi_D^T h_{44} - aG_D(t) \end{cases} . \tag{5}$$

Using the above, we have the following corollary about the corresponding direct adaptive control strategy the adaptive error system in 1:

Corollary: Assume the following:

1. There exists a gain, G_e^*such that the triple $(A_C \equiv A + BG_e^*C, B, C)$ is SPR (this is known to be equivalent to $CB > 0$ and the open loop transfer function

$$P(s) \equiv C(sI - A)^{-1}B \tag{6}$$

is minimum phase),
2. (3) is satisfied
3. Span(Γ) $\subseteq$ Span(B)

with a positive constants, then the output y exponentially approaches a neighborhood with radius proportional to the magnitude of the disturbance, v, for sufficiently small a and γ_i. Furthermore, each adaptive gain matrix is bounded.

This corollary provides a control law that is robust with respect to persistent disturbances and, exponentially with rate e^{-at}, produces:

$$\overline{\lim_{\tau\to\infty}} \|e(t)\| \leqslant \frac{\left(1+\sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} \|B\| \, M(\tau) \underset{t\to 0}{\to} 0.$$

Proof:

We form the Energy Storage Functions:

$$V = \frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}\mathrm{tr}\left[\Delta G\gamma^{-1}\Delta G^{\mathrm{T}}\right] \tag{7}$$

where $tr\,Q \equiv \sum\limits_{i=1}^{N} q_{ii}$ and $P > 0$ is the solution of the following pair of equations:

$$\begin{cases} A_c^T P + PA_c = -Q < 0 \\ PB = C^T \end{cases} \tag{8}$$

These equations are usually known as the Kalman-Yacubovic Conditions. The existence of a symmetric positive definite solution of (8) is known to be equivalent to the following condition:

$$T_c(s) \equiv C(sI - A_c)^{-1}B \tag{9}$$

strict positive realness (SPR). $T_C(s)$ (SPR) means, for some $\sigma > 0$,

$$\mathrm{Re}\, T_C(-\sigma + j\omega) \geqslant 0 \tag{10}$$

for all ω real. When the open-loop system (A, B, C) can be made SPR by output feedback $A_C \equiv A + BG_e^*C$, we say the open-loop system is almost strictly positive real (ASPR). This is known to be equivalent to $CB > 0$ and the open-loop $T(s) \equiv C(sI - A)^{-1}B$ being minimum phase, i.e. all transmission zeros stable; for example, see [16]. If we calculate the derivatives along the trajectories of (7), we have, using (1), that

$$\dot{V} = e^{\mathrm{T}} P A_c e + e^T P B w + e^T P \Delta f + \mathrm{tr}\left[\Delta \dot{G} \gamma^{-1} \Delta G^{\mathrm{T}}\right] + \nu^{\mathrm{T}} P e;$$

where

$$w \equiv \Delta G \eta$$

and

$$v \equiv B(u(t-\tau(t)) - u).$$

Invoking the equalities in the definition of SPR and substituting into the last expression, we get

$$\begin{cases} \dot{V} = -\frac{1}{2} e^{\mathrm{T}} Q e + \langle e_y, w \rangle + e^T P \Delta f - a \cdot \mathrm{tr}\left[G \gamma^{-1} \Delta G^{\mathrm{T}}\right] - \underbrace{tr(e_y z^T \Delta G^T)}_{\langle e_y, w\rangle} + \nu^{\mathrm{T}} P e \\ \le -\frac{1}{2} \underbrace{(q_{\min} - 2\mu_f p_{\max})}_{\beta} \|e\|^2 - a \cdot \mathrm{tr}\left[(\Delta G + G^*)\gamma^{-1} \Delta G^{\mathrm{T}}\right] + \nu^{\mathrm{T}} P e \\ \le -(\frac{1}{2}\beta \|e\|^2 + a \cdot \mathrm{tr}\left[\Delta G \gamma^{-1} \Delta G^{\mathrm{T}}\right]) + a \cdot \left|\mathrm{tr}\left[G^* \gamma^{-1} \Delta G^{\mathrm{T}}\right]\right| + \left|v^{\mathrm{T}} P e\right| \\ \le -(\frac{\beta}{2p_{\min}} e^T P e + 2a \bullet \frac{1}{2} \mathrm{tr}\left[\Delta G \gamma^{-1} \Delta G^{\mathrm{T}}\right]) + a \cdot \left|\mathrm{tr}\left[G^* \gamma^{-1} \Delta G^{\mathrm{T}}\right]\right| + \left|v^{\mathrm{T}} P e\right| \\ \le -2aV + a \cdot \left|\mathrm{tr}\left[G^* \gamma^{-1} \Delta G^{\mathrm{T}}\right]\right| + \left|v^{\mathrm{T}} P e\right| \end{cases}.$$

Now, using the Cauchy-Schwartz Inequality

$$\left|\mathrm{tr}\left[G^* \gamma^{-1} \Delta G^{\mathrm{T}}\right]\right| \le \|G^*\|_2 \|\Delta G\|_2$$

and

$$\left|v^{\mathrm{T}} P e\right| \le \left\|P^{\frac{1}{2}} \nu\right\| \left\|P^{\frac{1}{2}} e\right\| = \sqrt{v^T P \nu} \bullet \sqrt{e^T P e}$$

We will say that the unknown delay $\tau(t)$ is small when (3) is satisfied so,

$$\|\nu\| \equiv \|B\| \, \|u(t) - u(t-\tau(t))\| \leqslant \|B\| \, M(\tau).$$

We have

$$\begin{aligned} &\dot{V} + 2aV \leqslant a \cdot \|G^*\|_2 \|\Delta G\|_2 + \sqrt{p_{\max}} \, \|\nu\| \sqrt{e^T P e} \\ &\leqslant a \cdot \|G^*\|_2 \|\Delta G\|_2 + (\sqrt{p_{\max}} \|B\| \, M(\tau)) \sqrt{e^T P e} \\ &\leqslant (a\|G^*\|_2 + \sqrt{p_{\max}} \|B\| \, M(\tau)) \sqrt{2} \, \underbrace{[\frac{1}{2} e^T P e + \frac{1}{2} \, \|\Delta G\|_2^2]^{\frac{1}{2}}}_{V^{\frac{1}{2}}} \\ &\therefore \frac{\dot{V} + 2aV}{V^{\frac{1}{2}}} \leqslant (a\|G^*\|_2 + \sqrt{p_{\max}} \|B\| \, M(\tau)) \sqrt{2} \end{aligned}.$$

Now, using the identitytr $[ABC] = \mathrm{tr}\,[CAB]$,

$$\|G^*\|_2 \equiv [tr(G^*\gamma^{-1}(G^*)^T)]^{\frac{1}{2}} = [tr((G^*)^T G^*\gamma^{-1})]^{\frac{1}{2}}$$
$$\leq [(tr((G^*)^T G^*(G^*)^T G^*)^{\frac{1}{2}}(tr(\gamma^{-1}\gamma^{-1})^{\frac{1}{2}}]^{\frac{1}{2}}$$
$$= [tr(G^*(G^*)^T)]^{\frac{1}{2}}[tr\gamma^{-1}]^{\frac{1}{2}} \leq \frac{\|B\|M(\tau)}{aM_G} \bullet M_G = \frac{\|B\|M(\tau)}{a}$$

$$\Rightarrow \frac{\dot{V} + 2aV}{V^{\frac{1}{2}}} \leq (1 + \sqrt{p_{\max}})\|B\|\, M(\tau))\sqrt{2}, \tag{11}$$

from

$$\frac{d}{dt}(2e^{at}V^{\frac{1}{2}}) = e^{at}\frac{\dot{V} + 2aV}{V^{\frac{1}{2}}} \leq e^{at}(1 + \sqrt{p_{\max}})\|B\|\, M(\tau))\sqrt{2}.$$

Integrating this expression we have:

$$e^{at}V(t)^{1/2} - V(0)^{1/2} \leq \frac{\left(1 + \sqrt{p_{\max}}\right)\ \|B\|\, M(\tau)}{a}\left(e^{at} - 1\right)$$

$$\therefore V(t)^{1/2} \leq V(0)^{1/2}e^{-at} + \frac{\left(1 + \sqrt{p_{\max}}\right)\ \|B\|\, M(\tau)}{a}\left(1 - e^{-at}\right) \tag{12}$$

The function V is a norm function of the state e(t) and matrix G(t): so, since $V^{\frac{1}{2}}$ is bounded for all t, then e(t) and G(t) are bounded. We also have the following inequality:

$$\sqrt{p_{\min}}\,\|e(t)\| \leq V(t)^{1/2}.$$

Substitution of this into (12) gives us an exponential bound on state $e(\tau)$:

$$\|e(t)\| \leq \frac{e^{-at}}{\sqrt{p_{\min}}}V(0)^{1/2} + \frac{\left(1 + \sqrt{p_{\max}}\right)\ \|B\|\, M(\tau)}{a\sqrt{p_{\min}}}\left(1 - e^{-at}\right) \tag{13}$$

Taking the limit superior of (13), we have

$$\overline{\lim_{\tau\to\infty}}\,\|e(t)\| \leq \frac{\left(1 + \sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}}\|B\|\, M(\tau) \equiv R_* \tag{14}$$

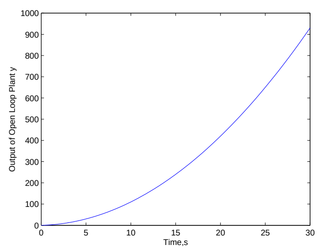
#

3 Simulation and Results

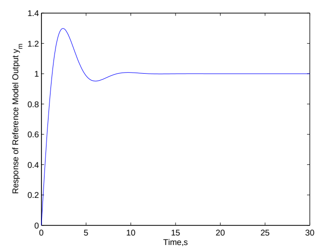
We will illustrate the above robust adaptive controller on the following plant:

$$\begin{cases} \dot{x} = \underbrace{\begin{bmatrix} x_2 \\ 0.3 * \sin(x_1) \end{bmatrix}}_{A(x)} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t-\tau) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\Gamma} u_D \\ y = \underbrace{\begin{bmatrix} 1 \ 0.1 \end{bmatrix}}_{C} x \end{cases} \quad (15)$$

We use step disturbances to provide simulation results for various small time varying values of delay $\tau(t)$. An adequate reference model must be developed for output tracking. The open loop output response to a step disturbance of magnitude 1 can be seen in Fig. 1(a). The desired reference model output, $y_m(t)$, for the closed loop reference model linear plant and lead controller to a step disturbance of magnitude 1 can be seen in Fig. 1(b). This reference model output was created by designing a lead controller to stabilize the plant and achieve the desired temporal response characteristics.



(a) Output response, $y(t)$

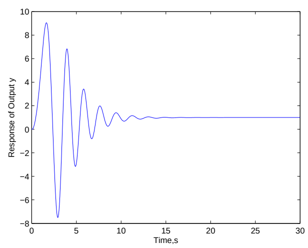


(b) Output response, $y_m(t)$

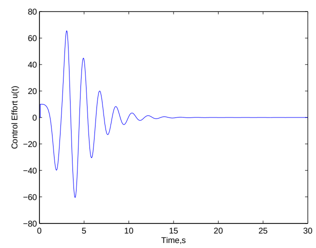
Fig. 1 Output response, (a) $y(t)$, for the open loop plant and (b) $y_m(t)$, for the closed loop reference model plant and lead controller to a step disturbance of magnitude 1.

3.1 Step Disturbances

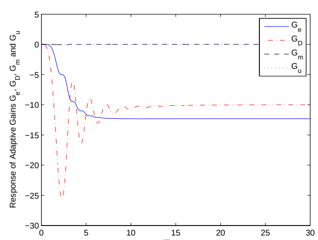
The response to a step disturbance of magnitude 10 of the output response, y(t), control effort u(t), and the adaptive gains for no input delay are shown in Fig. 2(a), 2(b) and 2(c) respectively.The response to a step disturbance of magnitude 10 of the output response, y(t), control effort u(t), and the adaptive gains for the input delay time, $\tau = 0.09s$ are shown in Fig. 3(a), 3(b) and 3(c) respectively. The response to a step disturbance of magnitude 10 of the output response, y(t), control effort u(t), and the adaptive gains for the input delay time, $\tau = 0.115s$ are shown in Fig. 4(a), 4(b) and 4(c) respectively. It can be seen that the adaptive error system adequately tracks the desired reference model output for the delay free system and the "small" delay case. As the constant delay grows, the adaptive system still tracks the desired reference model output, albeit with poor temporal characteristics. The waveform of time varying delay $\tau(t) = |0.56 * sin(10t) + 0.34|(s)$ is shown in Fig. 5(a). The response to a step disturbance of magnitude 10 of the output response, y(t), control effort u(t), and the adaptive gains for the input delay time, $\tau(t) = |0.56 * sin(10t) + 0.34|(s)$ are shown in Fig. 5(b), 5(c) and 5(d) respectively. This simulation has shown that the adaptive controller can force a simple midly-nonlinear plant to adequately track a linear reference model. The adaptive controller can operate in the presence of "small" constant and time varying delays without any knowledge of the delay.



(a) Output response, $y(t)$

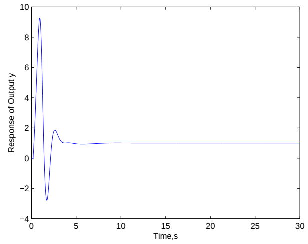


(b) Control Effort, u(t)

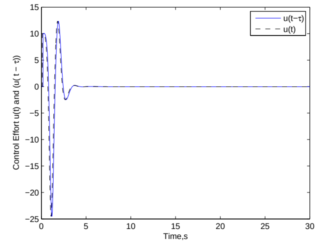


(c) Response of adaptive gains

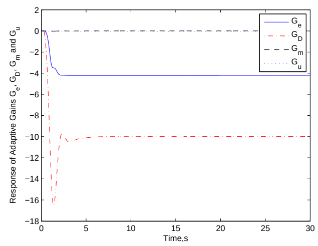
Fig. 2 (a) Output response, $y(t)$, (b) Control Effort, u(t) and (c) Response of adaptive gains for a step disturbance of magnitude 10 and no delay.



(a) Output response, $y(t)$

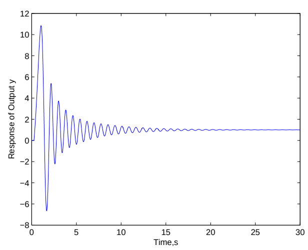


(b) Control Effort, u(t)

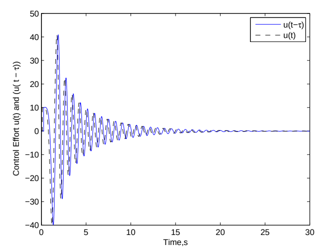


(c) Response of adaptive gains

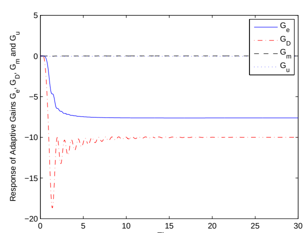
Fig. 3 (a) Output response, $y(t)$, (b) Control Effort, u(t) and (c) Response of adaptive gains for a step disturbance of magnitude 10 and $\tau = 0.09s$.



(a) Output response, $y(t)$

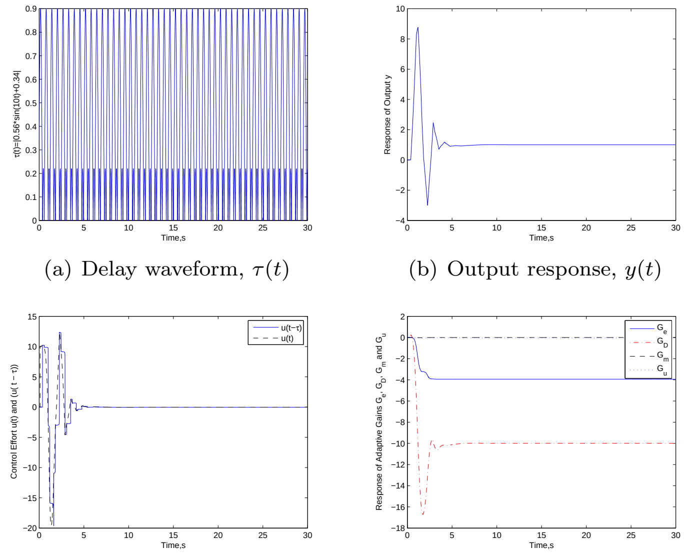


(b) Control Effort, u(t)



(c) Response of adaptive gains

Fig. 4 (a) Output response, $y(t)$, (b) Control Effort, u(t) and (c) Response of adaptive gains for a step disturbance of magnitude 10 and $\tau = 0.155s$.



(a) Delay waveform, $\tau(t)$

(b) Output response, $y(t)$

(c) Control Effort, u(t)

(d) Response of adaptive gains

Fig. 5 (a)Delay waveform, $\tau(t)$, (b) Output response, $y(t)$, (c) Control Effort, u(t) and (d) Response of adaptive gains for a step disturbance of magnitude 10 and $\tau(t) = |0.56 * sin(10t) + 0.34|$(s).

4 Conclusions

In this paper, a proof for the corollary developed for the Direct Model Reference Adaptive Tracking Control of mildly non-linear systems with unknown time varying input delays found in Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays - Part I was completed. The adaptive error system was developed for the DMRAC of mildly non-linear systems with unknown small constant or time varying input delays using the concept of un-delayed ideal trajectories. It has been shown that the adaptively controlled system is globally stable, but the adaptive tracking error is no longer guaranteed to approach the origin. However, exponential convergence to a neighborhood is achieved as a result of the control design. A simple example was provided to illustrate this adaptive control method.

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