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Estimation of Nonlinear Parameters from Simulated Data of an Aircraft

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Abstract The current paper discusses an improvement to well known Neural Gauss Newton(NGN) method, which makes the method capable of estimating nonlinear parameters from Flight data. The estimation is taken over for a set of simulated data with various control surface combinations. Then the estimation is carried out for the simulated data with selected control surface combination, for which noise is added, to test the handling capabilities of the Improved Neural Gauss Newton(INGN) method.

1 Introduction

System Identification is the process of providing a mathematical description to the system under consideration, from a set of measured quantities, for a given set of control inputs. We use the parameter estimation, which is an integral part of System Identification, for estimating the unknown parameters in the assumed mathematical model, based on the measurements made. Several methods have been proposed and implemented for the parameter estimation, but a very few methods have the ability of handling noise in a better way. Recently neural network based methods have been proposed to estimate aerodynamic parameters from flight data [9, 3]. Such approach generates a model based entirely on the input/output measurements of the system without trying to model the internal physical mechanism of the system. It is important to note that such an approach of model building for complex systems neither requires postulation of mathematical model nor the solving of equations of motion. Artificial neural networks(ANN) have been used to model aircraft dynamics where aircraft motion and control variables are mapped to predict the total aerodynamic

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coefficients [4, 6, 10]. In all these papers, the emphasis has been on aerodynamic modelling and estimation of aerodynamic coefficients using Feed Forward Neural Networks (FFNN). Raisinghani et al. [9, 3] proposed two new methods namely the Delta and the Zero method for explicitly estimating aircraft parameters from flight data using FFNN. Hence, the advantage of FFNNs in handling noise (having zero mean) in flight data to yield better and consistent estimates cannot be ignored. Further, the neural based methods have the advantage of being able to use a true nonlinear function as the working model for prediction.

2 Improved NGN Method [1]

The NGN method [8], proposed by Peyada et. al, serves as an excellent tool in recent times for parameter estimation from flight data. An improvement in NGN algorithm [1] has been tried out, which was proved to be worth of trying the same base algorithm for estimating nonlinear parameters. The finesse of the neural networks relies upon the choice of input vector given for training. If we need to improve the neural network, then it is best to start at the input vector. The better the training given to ANN, the better the prediction, for a wide range of control inputs. The estimation process we used is mainly based upon the angle of attack(α), pitch rate(q) and linear accelerations along X and Z axis(a_x and a_z). In NGN method, the input variables for training include the above mentioned. The force and moment coefficients are provided along with the accelerations, angle of attack(α), pitch rate(q) and pitch angle(θ) as outputs for training the neural model. The linear accelerations account mainly for the force coefficients, while the moment coefficient is characterized mainly based on pitch rate(q) and angle of attack(α). The data used for estimation is the trajectory model of the airplane under consideration i.e. time history of the variables. Since the neural model is developed based on the input and output vector, the idea of improving NGN is tried by including the first derivative of the important motion variables, in the longitudinal case $\dot{\alpha}$ and $\dot{q}$, for training. By introducing the time derivatives of angle of attack and pitch rate, ($\dot{\alpha}$ and $\dot{q}$), we can characterize the force and moment coefficients(C_L,C_D and C_m) better than NGN method as follows. Now by including these two vectors to the input matrix, the neural model will be able to predict the outputs much better. The pattern following ability of the neural network is exploited for this improvement. As we provide the first derivative of time derivatives of angle of attack and pitch rate, ($\dot{\alpha}$ and $\dot{q}$), the pattern following is improved, and hence the neural network. This improves the robustness of the neural model too, since we give the derivatives of the most influential variables for training. This improvement has brought better estimation of linear model with a much lower standard deviation. This is has been validated using the HANSA-3 flight data for longitudinal case, along with NGN for comparison [1]. The improvement made has been proved very effective for linear model. With this method, we can estimate nonlinear aerodynamic model. The input and output vector for the neural network is given below: The input vector at k^{th} instant is $U(k)$

$$U(k) = [\alpha(k), \theta(k), q(k), \dot{\alpha}(k), \dot{q}(k), C_D(k), C_L(k), C_m(k)] \tag{1}$$

where the $C_D(k)$, $C_L(k)$ and $C_m(k)$ at k^{th} instant are obtained using the measure quantity, $\alpha(k)$ in the following equations:

$$C_D(k) = -C_X(k) * \cos\alpha(k) - C_Z(k) \cdot \sin\alpha(k). \tag{2a}$$

$$C_L(k) = C_X(k) * \sin\alpha(k) - C_z(k) * \cos\alpha(k). \tag{2b}$$

$$\begin{aligned} C_m(k) = [&I_y * q(k) - I_{xz} * (p^2(k) - r^2(k)) \\ &- (I_z(k) - I_x(k)) * p(k)r(k) \\ &- T_{eng} * \cos\sigma_e ng * Z_{eng} - T_{eng} * \sin\sigma_e ng * X_{eng}]/(\overline{q}(k)S\overline{C}). \end{aligned} \tag{2c}$$

and C_X(k) and C_Z(k) are computed as below.

$$C_X(k) = \frac{m * a_{X_{CG}}(k)}{\overline{q}(k)S}. \tag{3a}$$

$$C_Z(k) = \frac{m * a_{Z_{CG}}(k)}{\overline{q}(k)S}. \tag{3b}$$

where

- p, q, r are the angular rates about the three axis respectively
- σ_{eng} is the engine thrust setting angle of the airplane
- Z_{eng} and X_{eng} are the offset distances from the CG
- T_{eng} is the engine thrust

The output vector for the neural network training is organized as follows:

$$Z(k+1) = [\alpha(k+1), \theta(k+1), q(k+1), \dot{q}(k+1), a_x(k+1), a_z(k+1)]. \tag{4}$$

3 Data generation for Estimation Purposes

A five degree-of-freedom (DOF) rig system has been used to simulate near free flight manoeuvres inside the wind tunnel by exciting a scaled down model aircraft through various types of control surface deflections. The equations of motion are presented by Peyada.N.K. et.al [7]. We solve the equations of motion of the 5 DOF dynamic rig for the given speed and control surface deflection to simulate the data [2] for the McDonnell Douglas F-4 aircraft. For longitudinal case, the derivative values, which also include the cross derivatives, are given in the tables 1 and 2. Hence, the data required for Estimation process requires the inclusion of trajectory variable angle of side slip. To generate such data, we have to simulate the data by deflecting both elevator and rudder. By deflecting both elevator and rudder, the combined effect of angle of attack and angle of sideslip is captured in the data, which is also done for four different combinations of the control surfaces which maximizes the energy of the whole input [5]. The simulation is carried out for two Angle of attack regimes, i.e. $\alpha \leq 15^0$ and $15^0 < \alpha \leq 30^0$.

Table 1: Original values of derivatives for $\alpha \leq 15^0$[2]

Derivative	**Original Value**	**Derivative**	**Original Value**	**Derivative**	**Original Value**
C_{X_0}	-4.34E-02	C_{Z_0}	-1.31E-01	C_{m_0}	-6.61E-03
C_{X_α}	2.39E-03	C_{Z_α}	-5.38E-02	C_{m_α}	-2.67E-03
$C_{X_{\beta^2}}$	2.53E-05	$C_{Z_{\delta_e}}$	-4.76E-03	$C_{m_{\beta^2}}$	-6.48E-05
$C_{X_{\alpha\beta^2}}$	-1.07E-06	$C_{Z_{\delta_e\alpha}}$	-3.30E-05	$C_{m_{\alpha\beta^2}}$	-2.65E-06
$C_{X_{\delta_e}}$	9.50E-04	C_{Z_q}	-6.36	$C_{m_{\delta_e}}$	-6.54E-03
$C_{X_{\delta_e\beta^2}}$	-8.50E-07	$C_{Z_{q\alpha}}$	2.96E-01	$C_{m_{\delta_e\alpha}}$	-8.49E-05
C_{X_q}	5.00E-01	$C_{Z_{q\alpha^2}}$	6.30E-02	$C_{m_{\delta_e\beta^2}}$	3.74E-06
$C_{X_{q\alpha}}$	5.73E-02			C_{m_q}	-2.71
$C_{X_{q\alpha^2}}$	-1.00E-02			$C_{m_{q\alpha}}$	-9.00E-02

Table 2: Original values of derivatives for $15^0 < \alpha \leq 30^0$[2]

Derivative	**Original Value**	**Derivative**	**Original Value**	**Derivative**	**Original Value**
C_{X_0}	1.41E-01	C_{Z_0}	-6.08E-01	C_{m_0}	5.490E-02
C_{X_α}	-1.54E-02	C_{Z_α}	-2.20E-02	C_{m_α}	-6.080E-03
$C_{X_{\alpha^2}}$	2.96E-04	$C_{Z_{\delta_e}}$	-6.77E-03	$C_{m_{\beta^2}}$	-1.690E-04
$C_{X_{\beta^2}}$	-3.72E-04	$C_{Z_{\delta_e\alpha}}$	9.7E-5	$C_{m_{\alpha\beta^2}}$	5.640E-07
$C_{X_{\alpha\beta^2}}$	4.14E-5	C_{Z_q}	1.136	$C_{m_{\delta_e}}$	-8.140E-03
$C_{X_{\alpha^2\beta^2}}$	-9.1E-7	$C_{Z_{q\alpha}}$	-1.418E-01	$C_{m_{\delta_e\alpha}}$	1.100E-04
$C_{X_{\delta_e}}$	1.82E-03	$C_{Z_{q\alpha^2}}$	3.110E-03	C_{m_q}	-9.510E-02
$C_{X_{\delta_e\alpha}}$	-7.3E-5			$C_{m_{q\alpha}}$	1.400E-03
C_{X_q}	-6.02E-02				
$C_{X_{q\alpha}}$	2.04E-03				

4 I-NGN Estimation

The I-NGN method is applied for the above mentioned simulated data individually, to obtain the derivatives. The trained and estimated trajectory are shown in the figures 1 and 2. The results are compared in the error plot comparing with the

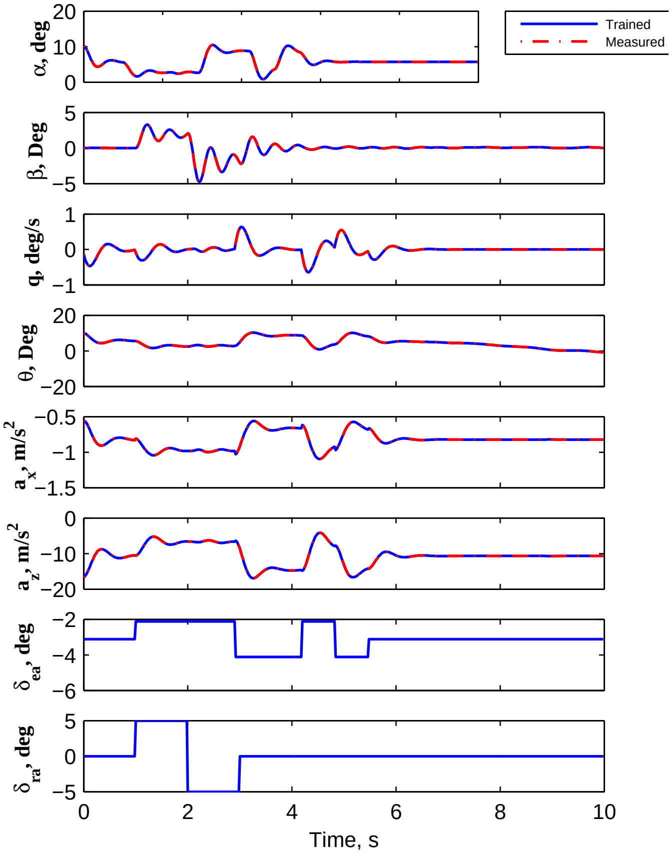

Fig. 1: Data without noise - Training

original values, which are given the table 1. The error plot(Figure 3) shows that the combination of $\delta_e(\pm 2^0)$ with 3211 configuration and $\delta_r(\pm 5^0)$ with Doublet configuration, is behaving well enough to estimate the parameters close to the originals, even though the values of the derivatives are very small. In case of strong derivatives such as, C_{z_α} and C_{m_α}, the error is very small i.e. well within 1%. Even for the cross derivatives the error with respect to the original values is within 3%. This proves that the method is very effective in estimating the nonlinear and coupled derivatives irrespective of the tininess of the value. But this is for the case of simulation data, which doesn't have real measurement noise or other noises. Now to apply this method in the real flight data, we should verify the robustness of the I-NGN method to different level of noises. The derivatives of coefficients are numbered and corresponding

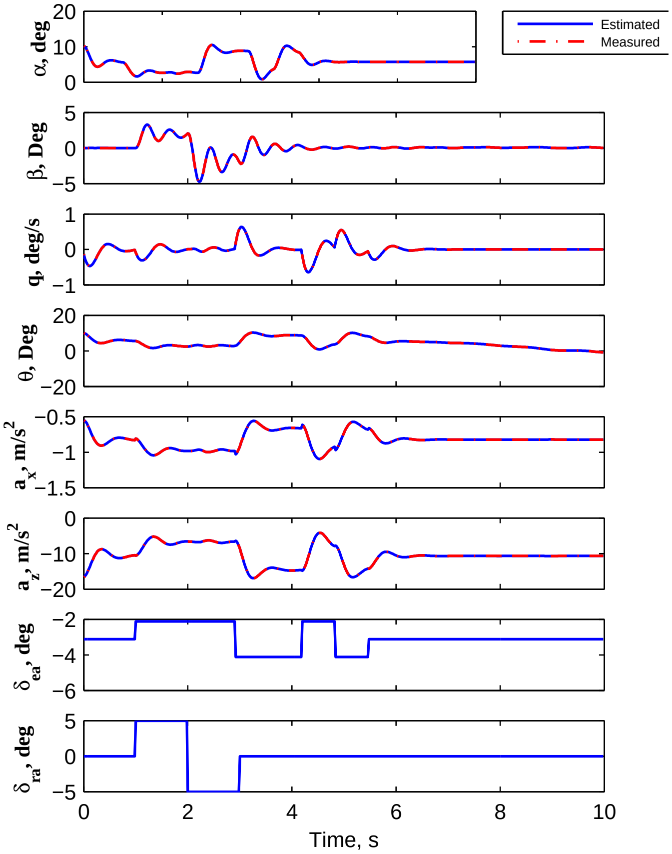

Fig. 2: Data without noise - Estimation

numbers are used for plotting the error of individual derivatives. The numbering is explained in the caption below the figure 3.

5 Noise Introduction

The noise is introduced as a Gaussian noise model to the all trajectory variables, as the following equation depicts. The noise was introduced for four different scales from 0.01 to 0.2.

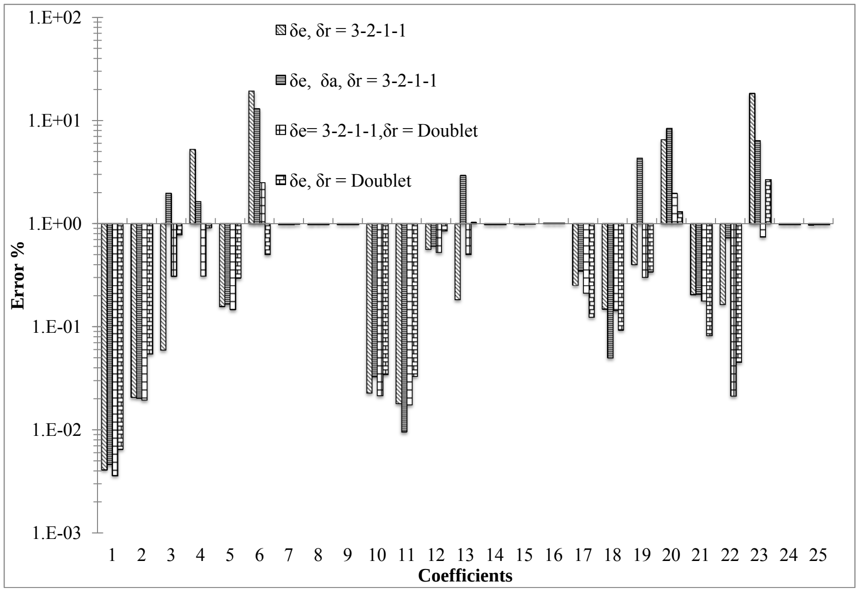

Fig. 3: Error Plot for different set of Control Surface Deflections
(1 : C_{X_0}, 2 : C_{X_α}, 3 : $C_{X_{\beta 2}}$, 4 : $C_{X_{\alpha\beta 2}}$, 5 : $C_{X_{\delta_e}}$, 6 : $C_{X_{\delta_e\beta 2}}$, 7 : C_{X_q}, 8 : $C_{X_{q\alpha}}$, 9 : $C_{X_{q\alpha 2}}$,
10 : C_{Z_0}, 11 : C_{Z_α}, 12 : $C_{Z_{\delta_e}}$, 13 : $C_{Z_{\delta_e\alpha}}$, 14 : C_{Z_q}, 15 : $C_{Z_{q\alpha}}$, 16 : $C_{Z_{q\alpha 2}}$,
17 : C_{m_0}, 18 : C_{m_α}, 19 : $C_{m_{\beta 2}}$, 20 : $C_{m_{\alpha\beta 2}}$, 21 : $C_{m_{\delta_e}}$, 22 : $C_{m_{\delta_e\alpha}}$, 23 : $C_{m_{\delta_e\beta 2}}$,
24 : C_{m_q}, 25 : $C_{m_{q\alpha}}$)

$$Z_n = Z + n' * Z * scale; \tag{5}$$

where Z is the set of simulated data, Z_n is the noise added data, n is vector of random numbers between 0 and 1.

5.1 Low Angle Regime

The results for low angles of attack ($\alpha \leq 15^0$) are shown in figures 5, 6 and 7. For strong derivatives, the noise introduction doesn't bring significant changes in the estimation process. The I-NGN is capable of handling noise level in the simulated measurement vector well enough such that, the error is well within 10% even for the noise scale of 0.2 of the simulated variables. Thus the method holds good amount of robustness for noises. While checking the error level for the cross derivatives and nonlinear derivatives, the most affected derivative is the one which the square of angle of sideslip(β^2), the deviation goes very high for the highest scale i.e. 0.2. But for lower noise levels the error deviation is still under control. The lower numerical

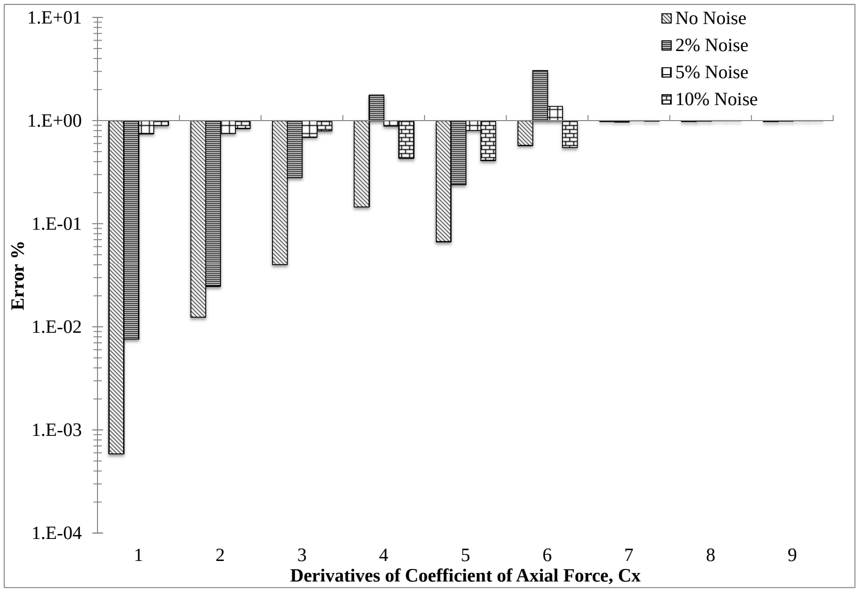

Fig. 4: Error Plot in $\alpha \leq 15^0$ - Axial Force Coefficient C_X
$(1 : C_{X_0}, 2 : C_{X_\alpha}, 3 : C_{X_{\beta^2}}, 4 : C_{X_{\alpha\beta^2}}, 5 : C_{X_{\delta_e}}, 6 : C_{X_{\delta_e\beta^2}}, 7 : C_{X_q}, 8 : C_{X_{q\alpha}}, 9 : C_{X_{q\alpha^2}})$

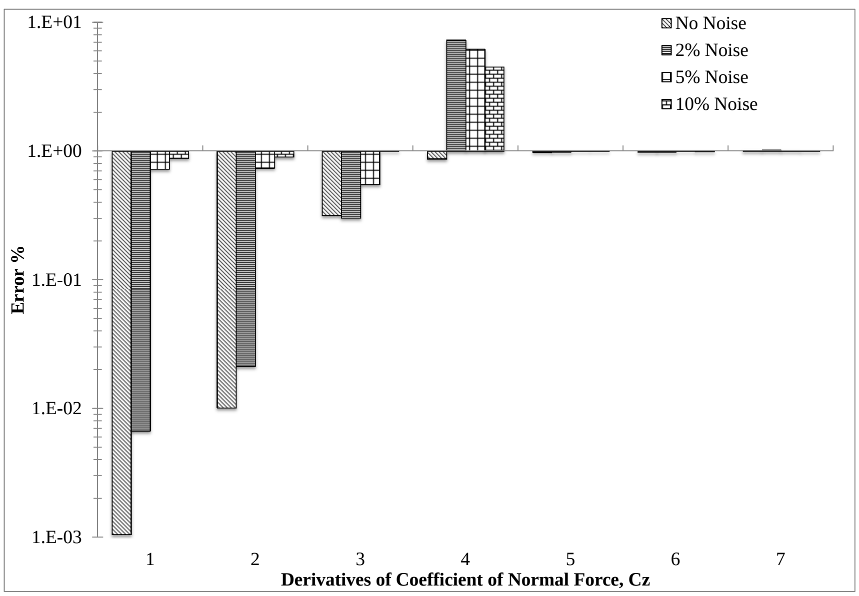

Fig. 5: Error Plot $\alpha \leq 15^0$ - Normal Force Coefficient C_Z
$(1 : C_{Z_0}, 2 : C_{Z_\alpha}, 3 : C_{Z_{\delta_e}}, 4 : C_{Z_{\delta_e\alpha}}, 5 : C_{Z_q}, 6 : C_{Z_{q\alpha}}, 7 : C_{Z_{q\alpha^2}})$

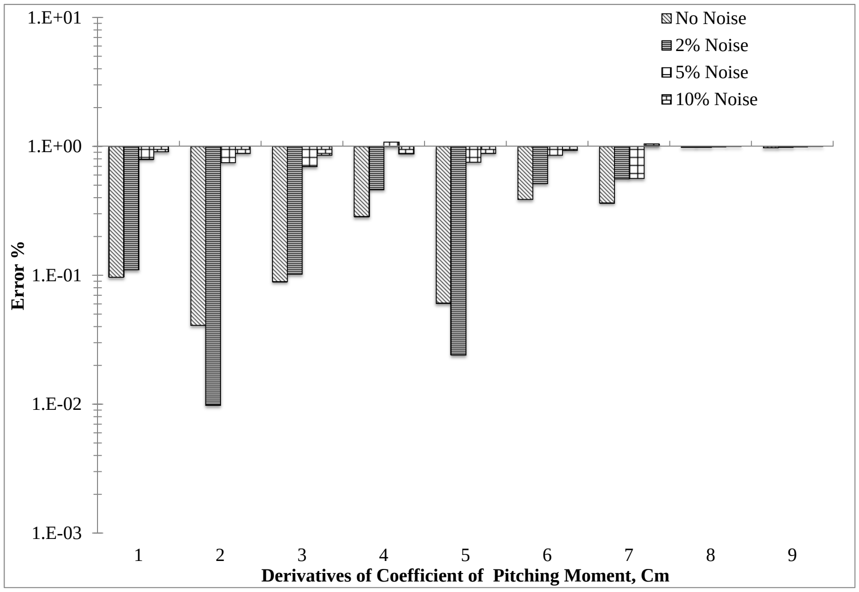

Fig. 6: Error Plot $\alpha \leq 15^0$ - Pitching Moment Coefficient C_m $(1 : C_{m_0}, 2 : C_{m_\alpha}, 3 : C_{m_{\beta^2}}, 4 : C_{m_{\alpha\beta^2}}, 5 : C_{m_{\delta_e}}, 6 : C_{m_{\delta_e\alpha}}, 7 : C_{m_{\delta_e\beta^2}}, 8 : C_{m_q}, 9 : C_{m_{q\alpha}})$

values for the derivatives may be a good reason for this deviation, since even a considerable amount noise may bring out big change in the derivatives. The order of magnitude analysis will say that the values are being deviated largely from the original value, which is very small. The other nonlinear derivatives have smaller error compared to the derivatives having angle of sideslip(β). Since most of the nonlinear derivatives are very small compared to the strong, linear derivatives, we need to have the range of data for higher values of these derivatives.

5.2 High Angle regime

The same estimation is carried out for the angle of attack range of $15^0 - 30^0$, for the same noise scales, as before. Most of the parameters are the same, except for a few which appear only in this regime. The error plots are given in figures 7, 8 and 9. The effect of noise is seen in all the derivatives. For some nonlinear parameters, the error is very high, the reason being the small value of these parameters when compared to other parameters. In high alpha regime, the smallest change in variables due to noise may result in considerable deviation of the parameters during the estimation, hence the higher error percentage. The linear parameters, which includes some

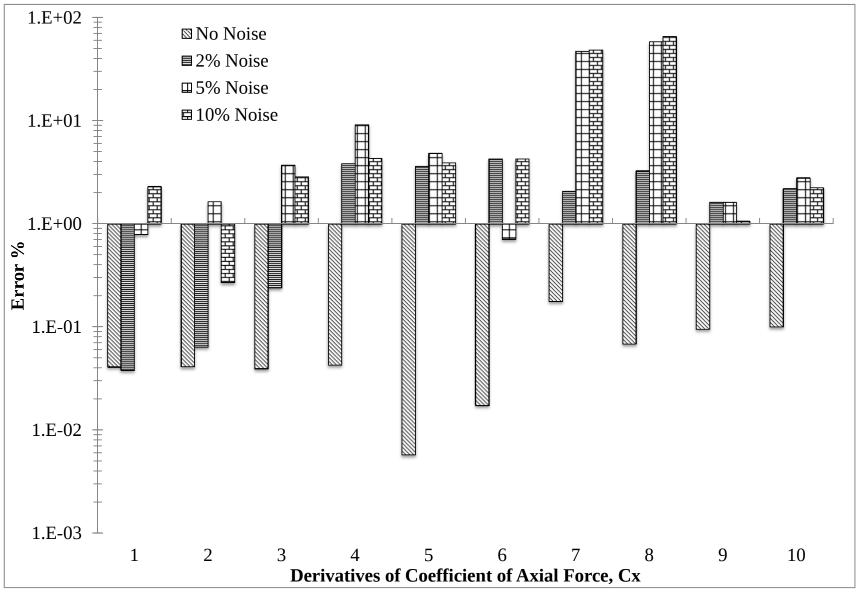

Fig. 7: Error Plot in $15^0 < \alpha \leq 30^0$ - Axial Force Coefficient C_X
($1 : C_{X_0}$, $2 : C_{X_\alpha}$, $3 : C_{X_{\alpha^2}}$, $4 : C_{X_{\beta 2}}$, $5 : C_{X_{\alpha\beta^2}}$, $6 : C_{X_{\alpha^2\beta^2}}$, $7 : C_{X_{\delta_e}}$, $8 : C_{X_{\delta_e\alpha}}$, $9 : C_{X_q}$,
10: $C_{X_{q\alpha}}$)

strong derivatives such as, $C_{L\alpha}, C_{m\alpha}, C_{m_{\delta_e}}$, are estimated well enough even in higher noise scales. This method is efficient and effective in high angle regimes too, with moderate noise levels. But for the nonlinear parameters, the method is less effective when the noise levels are increased. But still, if we can filter the noises out as much as possible, then the method is well effective in its purpose. This is evident from the comparison plots figures 10 and 11. The first plot compares the linear derivatives, with zero noise level, for two angle of attack regimes under study. From the plot, it is evident that these parameters are estimated well enough in both regimes, with the error from their original values is less than 1%, barring one parameter($C_{Z_{\delta_e}}$). The second plot compares the nonlinear derivatives which are common for the two angle regimes. These nonlinear parameters are very small compared to the linear derivatives,but still the method is capable of estimating mos of them even in high angle regimes with error less than 1%. Now this gives us a good proof, that the method is capable of handling nonlinear estimation.

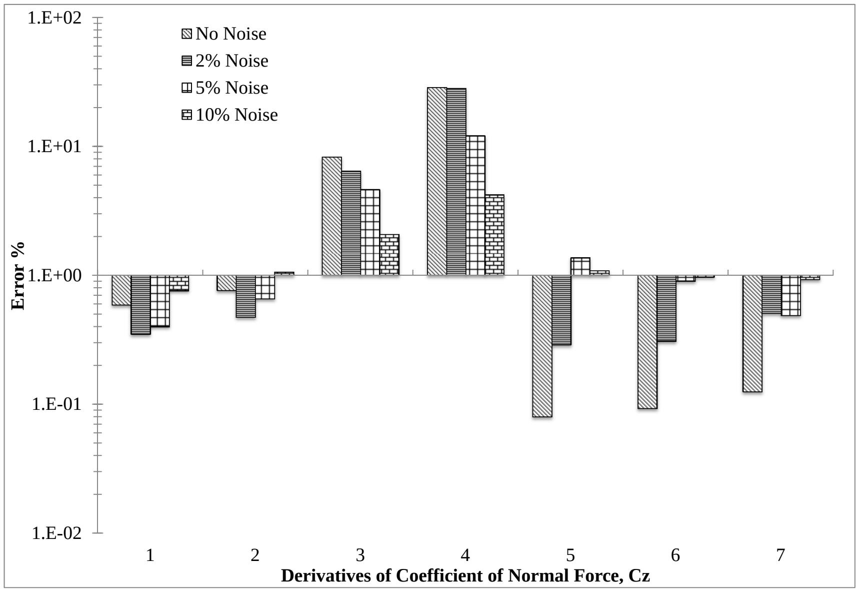

Fig. 8: Error Plot in $15^0 < \alpha \leq 30^0$ - Normal Force Coefficient C_Z
$(1 : C_{Z_0}, 2 : C_{Z_\alpha}, 3 : C_{Z_{\delta_e}}, 4 : C_{Z_{\delta_e\alpha}}, 5 : C_{Z_q}, 6 : C_{Z_{q\alpha}}, 7 : C_{Z_{q\alpha^2}},)$

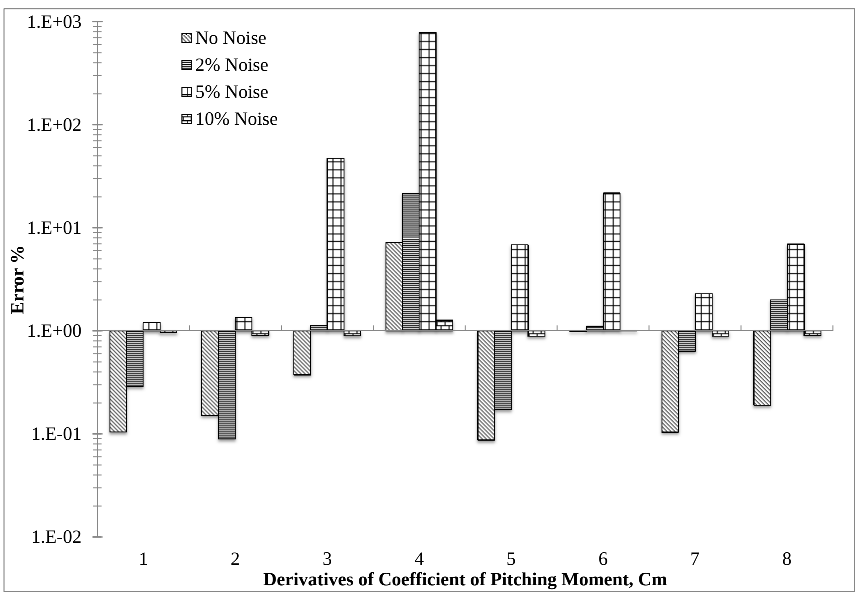

Fig. 9: Error Plot in $15^0 < \alpha \leq 30^0$ - Pitching Moment Coefficient C_m
$(1 : C_{m_0}, 2 : C_{m_\alpha}, 3 : C_{m_{\beta^2}}\ , 4 : C_{m_{\alpha\beta^2}}, 5 : C_{m_{\delta_e}}, 6 : C_{m_{\delta_e\alpha}}, 7 : C_{m_q}, 8 : C_{m_{q\alpha}})$

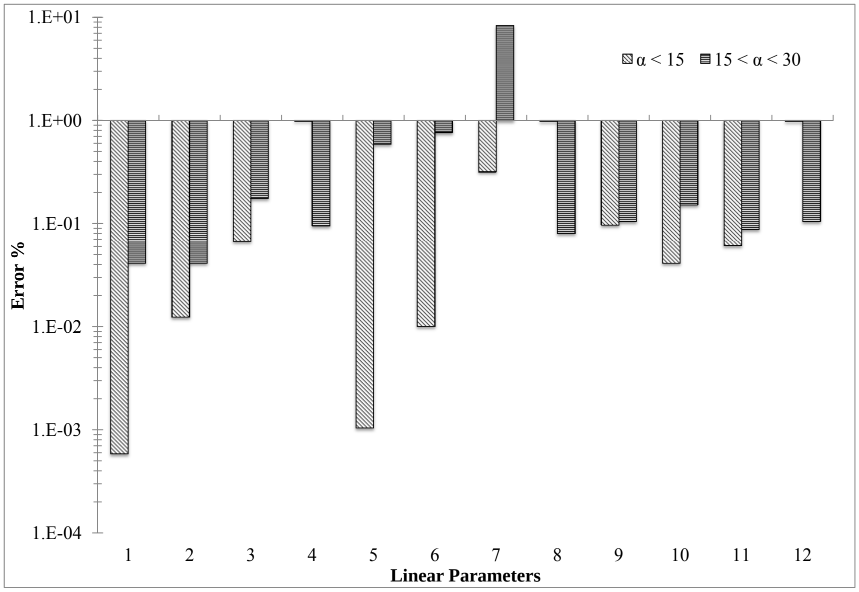

Fig. 10: Comparison plot for Linear Parameters
($1: C_{X_0}$, $2: C_{X_\alpha}$, $3: C_{X_{\delta_e}}$, $4: C_{X_q}$, $5: C_{Z_0}$, $6: C_{Z_\alpha}$, $7: C_{Z_{\delta_e}}$, $8: C_{Z_q}$, $9: C_{m_0}$, $10: C_{m_\alpha}$, $11: C_{m_{\delta_e}}$, $12: C_{m_q}$)

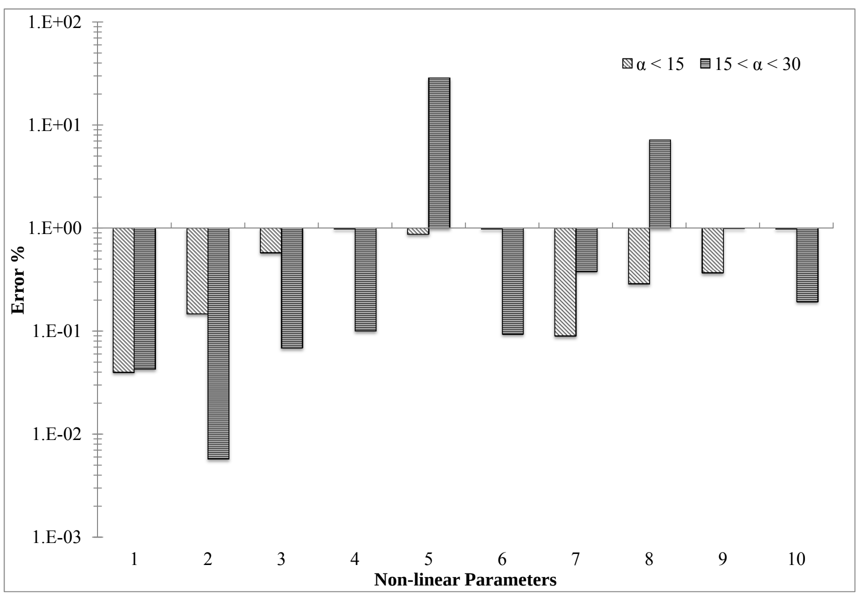

Fig. 11: Comparison plot for nonlinear Parameters
($1: C_{X_{\beta^2}}$, $2: C_{X_{\alpha\beta^2}}$, $3: C_{X_{\delta_e\beta^2}}$, $4: C_{X_{q\alpha}}$, $5: C_{Z_{\delta_e\alpha}}$, $6: C_{Z_{q\alpha}}$, $7: C_{m_{\beta^2}}$, $8: C_{m_{\alpha\beta^2}}$, $9: C_{m_{\delta_e\beta^2}}$, $10: C_{m_{q\alpha}}$)

6 Conclusion

The improved NGN has been tried out for simulated longitudinal trajectory data of F-4 [2] aircraft. The simulation is carried out for four combinations of Control surface deflections, from which one combination is selected for estimation with noises. The noise is introduced for four scales, which is introduced in all measured variables. The estimation is carried out with noisy data and the error plot is presented. The strong parameters are not affected much by the noises, but some nonlinear parameters are affected in a large scale. The Noise level increase may pose a big threat to the method, but never-the-less, the method is effective and efficient for high angle regimes, where the nonlinearity plays a big role. This effectiveness has to be tested with the real flight nonlinear data, without which this method may not be good for nonlinear estimation. This method gives a good starting point for the better exploration of nonlinear regimes for any unknown system.

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