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Incremental Backstepping for Robust Nonlinear Flight Control

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Abstract This paper presents a robust nonlinear flight control strategy based on results combining incremental control action and the backstepping design methodology for vehicles described by strict-feedback (cascaded) nonlinear systems. The approach, referred to as incremental backstepping, uses feedback of actuator states and acceleration estimates to allow the design of increments of control action. In combination with backstepping, the proposed approach stabilizes or tracks outer-loop control variables of the nonlinear system *incrementally*, accounting for large model and parametric uncertainties, besides undesired factors such as external perturbations and aerodynamic modeling errors. With this result, dependency on the modeled aircraft system is greatly reduced, overcoming the major robustness flaw of conventional model-based flight control strategies. This suggested methodology implies a trade-off between accurate knowledge of the dynamic model and accurate knowledge of the vehicle sensors and actuators, which makes it more suitable for practical application than identification or model based adaptive control architectures. Simulation results verify the tracking capability and superior robustness of the proposed controller under aerodynamic uncertainty with respect to standard backstepping methodologies for a simple flight control example.

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1 Introduction

The design of a generic robust nonlinear flight control strategy is considered in this paper. The strategy is based on recent results combining incremental control action and the backstepping design methodology for strict-feedback (cascaded) nonlinear systems, called incremental backstepping. The main design issue is dealing with large model and parametric uncertainties present in flight control systems, mainly because of aerodynamic and unmodeled dynamics.

Incremental backstepping is presented by means of a modification to the standard backstepping design methodology that reduces its dependency on the baseline aircraft model, through the use of actuator states and acceleration estimates. These considerations allow the design of increments of control action which, in combination with backstepping, helps to stabilize or track outer-loop control variables of the nonlinear system incrementally. In contrast to regular backstepping, this method is inherently *implicit* in the sense that desired closed-loop dynamics do not reside in some explicit model to be cancelled, but which results when the feedback loops are closed.

Theoretical development of increments of nonlinear control action date back from the late nineties and started with activities concerning 'Implicit Dynamic Inversion' for DI-based flight control [24, 4], where the architectures considered in this paper were firstly described. Other designations for these developments found in the literature are 'Modified NDI' and 'Simplified NDI', but the designation 'Incremental NDI' is considered to describe the methodology and nature of these type of control laws better [9, 10, 11, 21]. INDI has been elaborated and applied theoretically in the past decade for flight control and space applications [21, 25, 4, 5, 6, 1].

The main motivation of this approach is to bring the implicitness of such sensor-based architectures with Lyapunov-based controller design such as backstepping for aerospace applications. This topic has been introduced in the literature recently, but from a singular pertubations approach, in [14]. The recursive step-by-step procedure of the backstepping methodology can be exploited for the design of a single and generic control law for cascaded systems, retaining by definition its stability and convergence properties, and with the possibility to retain stabilizing nonlinearities in the closed-loop system description.

The remainder of the paper is organized as follows. Section 2 presents the main results of this paper, namely the incremental backstepping approach. In Section 3 we present the generic flight control law design with this method and for the particular case of attitude control. Section 5 illustrates the design of incremental backstepping control for an examplary longitudinal missile tracking control, including simulations of such control strategy. Conclusions are provided in Section 6.

2 Incremental Backstepping

This section presents the proposed incremental backstepping approach. Its design departure is from a stability and convergence viewpoint due to *control Lyapunov function* augmentations rather than forcing linear behaviour through conventional feedback linearization. Because of its advantage of stabilizing or tracking one or more loops within a single control command maintaining desired properties, the motivation for this approach also stems to the combined flexibility of this method over conventional approaches such as robust nonlinear dynamic inversion (NDI) [2, 3, 7, 12, 13, 15, 22, 23, 26, 27], and its adaptive [8, 19, 20] and incremental counterparts [1, 4, 5, 6, 9, 21, 24, 25].

For the discussion, we will consider physical systems or vehicle dynamics which are represented by the following strict-feedback second order cascaded form:

$$\dot{\xi} = \mathbf{h}(\xi) + \mathbf{k}(\xi)\mathbf{x} \tag{1a}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\xi, \mathbf{x}) + \mathbf{G}(\xi, \mathbf{x})\mathbf{u} \tag{1b}$$

We assume that Eq. (1a) may represent a kinematic equation, i.e., a relation between (angular) velocities and positions (orientations), while Eq. (1b) may represent a dynamic equation relating forces and torques to the former (angular) velocities, see Figure 1. In flight control, Eq. (1a) may also have a control input dependency, if not always, but this term is ignored during the control design of the kinematic loop since the backstepping method can only handle nonlinear systems of lower-triangular form (e.g., for attitude control the assumption is made that the fin surface is a pure moment generator). Although this method is presented for second-order strict feedback (cascaded) nonlinear systems, its extension to higher-order systems by continuation of the backstepping design methodology is straightforward. This is of particular interest, if for instance, several control loops are to be considered for the control law design (e.g., position control, etc.).

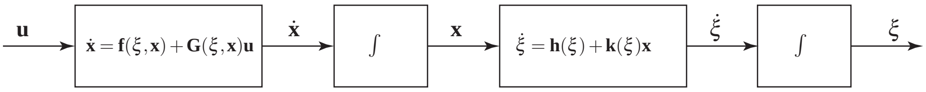

Fig. 1 Cascade structure of the system in Eqs. (1).

The closed-loop stability of the complete system for this cascaded interconnection will rely on the efficient design of a control law $\mathbf{u}$. We start the discussion with a brief review of the backstepping (denoted 'BKS') procedure [17, 18] for stabilization, in this case as follows:

Step 1

1. Promoting $\mathbf{x}$ as the virtual control in Eq. (1a), introduce the error state as:

$$\mathbf{z} = \mathbf{x} - \mathbf{x}_{\text{des}} = \mathbf{x} - \alpha(\xi)$$

   where $\alpha(\xi)$ is a stabilizing feedback that will be designed in the following sub-steps. Such intermediate control law is referred as a *stabilizing function*. Rewriting Eq. (1a) in terms of this error state results in:

$$\dot{\xi} = \mathbf{h}(\xi) + \mathbf{k}(\xi)\,(\mathbf{z} + \alpha)$$

2. Construct any positive definite, radially unbounded function $V_1(\xi) : \mathbf{R}^3 \mapsto \mathbf{R}^+$ as a *control Lyapunov function* (CLF) for the system, treating it as a final stage, e.g.,

$$V_1(\xi) = \frac{1}{2}\xi^\top \xi$$

   This choice of a CLF may depend on the kinematic equation considered and may trade-off its complexity with the resulting control law.
3. To find a stabilizing function $\alpha(\xi)$ for the virtual control in this step ($\mathbf{x}$), we need to make the derivative of $V_1(\xi)$ nonpositive when $\mathbf{x} = \alpha$. Such continuously differentiable feedback control law $\alpha(\xi)$ hence need to satisfy:

$$\dot{V}_1 = \frac{\partial V_1(\xi)}{\partial \xi}\Big[\mathbf{h}(\xi) + \mathbf{k}(\xi)\alpha(\xi)\Big] \leq -W(\xi) \leq 0, \quad \forall \xi \in \mathbf{R}^n$$

   where $W : \mathbf{R}^n \mapsto \mathbf{R}$ is positive semi-definite. Moreover, for the subsequent steps, the following notation for the derivative of the current stabilizing function $\alpha(\xi)$ is introduced:

$$\dot{\alpha}(\xi, \mathbf{x}) = \frac{\partial \alpha(\xi)}{\partial \xi}\dot{\xi} = \frac{\partial \alpha(\xi)}{\partial \xi}\Big[\mathbf{h}(\xi) + \mathbf{k}(\xi)\Big(\mathbf{z} + \alpha(\xi)\Big)\Big]$$

Step 2

This step consists of calculating the final control law $\mathbf{u}$ as follows.

1. With $\alpha(\xi)$ determined, the next step is to consider the subsequent state equation, the dynamics in Eq. (1b), in terms of the error state:

$$\dot{\mathbf{z}} = \dot{\mathbf{x}} - \dot{\alpha}(\xi, \mathbf{x}) = \mathbf{f}(\xi, \mathbf{x}) + \mathbf{G}(\xi, \mathbf{x})\mathbf{u} - \dot{\alpha}(\xi, \mathbf{x})$$

2. Construct an augmented CLF for the system, treating it as a final stage:

$$V_2(\xi, \mathbf{x}) = V_1 + \frac{1}{2}\mathbf{z}^\top \mathbf{z}$$

3. To find the final control law $\mathbf{u}$ in this step, we need to make the derivative of $V_2(\xi,\mathbf{x})$ nonpositive when $\xi \neq \alpha$

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \mathbf{z}^\top \dot{\mathbf{z}} \\ &\leq -W(\xi) + \mathbf{z}^\top \left[\mathbf{f}(\xi,\mathbf{x}) + \mathbf{G}(\xi,\mathbf{x})\mathbf{u} - \dot{\alpha}(\xi,\mathbf{x}) + \frac{\partial V_1(\xi)}{\partial \xi}\mathbf{k}(\xi)\right]\end{aligned}$$

If $\mathbf{G}(\xi,\mathbf{x}) \neq 0$ and invertible for all $\mathbf{x}$ and ξ, *one* possible choice for $\mathbf{u}$ is:

$$\mathbf{u} = \mathbf{G}^{-1}(\xi,\mathbf{x})\left[-c_1\mathbf{z} - \mathbf{f}(\mathbf{x}) + \dot{\alpha}(\xi,\mathbf{x}) - \frac{\partial V_1(\xi)}{\partial \xi}\mathbf{k}(\xi)\right] \tag{2}$$

with $c_1 > 0$, which yields $\dot{V}_2 \leq -W(\xi) - c_1\mathbf{z}^\top\mathbf{z} \leq 0$. However, as we pointed out before, many other, possibly better, choices for α could be available, even if $\mathbf{G}(\xi,\mathbf{x}) = 0$ at some points.

It should be clear that this result of backstepping for cascaded second order systems is not the specific form of the control law (2), but rather the construction of a stabilizing function for the kinematic equation that depends on the choice of a Lyapunov function whose derivative can be made negative by a wide variety of family of control laws. Also, the augmentation of this selected Lyapunov function in the second step may have other structure, which could result in a different family of controllers. This flexibility in backstepping gives a great advantage to the control engineer, in which the complexity of the CLFs can be traded with the complexity of the resulting controller structure. This backstepping procedure can be illustrated as in Figure 2.

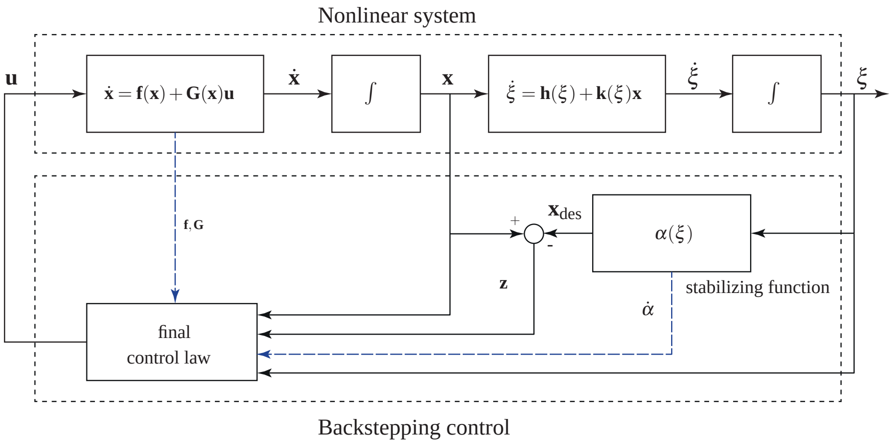

Fig. 2 Backstepping control block diagram for second order cascaded systems. Dashed arrows represent information required for control design. Notice that the final control law requires knowledge of both **f** and **G**.

The incremental backstepping (denoted 'IBKS') is derived from expressing or approximating the dynamics into an incremental form. This incremental form of the dynamic equation is obtained as follows [4]. Consider a generic form of an affine nonlinear dynamical system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \tag{3}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the state vector, $\mathbf{u} \in \mathbf{R}^m$ is the control input vector, $\mathbf{f}$ and $\mathbf{h}$ are smooth vector fields on $\mathbf{R}^n$, and $\mathbf{G} \in \mathbf{R}^{n\times m}$ is a matrix whose columns are smooth vector fields $\mathbf{g}_j$. A standard Taylor series expansion provides the following first-order approximation of $\dot{\mathbf{x}}$ for $\mathbf{x}$ and δ in the neighborhood of $[\mathbf{x}_0, \mathbf{u}_0]$:

$$\dot{\mathbf{x}} \cong \mathbf{f}(\mathbf{x}_0) + \mathbf{G}(\mathbf{x}_0)\mathbf{u}_0 + \frac{\partial}{\partial \mathbf{x}}\left[\mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}\right]\Big|_{\substack{\mathbf{x}=\mathbf{x}_0\\ \mathbf{u}=\mathbf{u}_0}} (\mathbf{x} - \mathbf{x}_0) + \mathbf{G}(\mathbf{x}_0)\,(\mathbf{u} - \mathbf{u}_0) + H.O.T. \tag{4}$$

where the current state and control, $\mathbf{x}_0$ and $\mathbf{u}_0$ respectively, represent for each time instance the *reference* an incremental instance in time before $\mathbf{x}$ and $\mathbf{u}$ for the construction of the first-order approximation of $\dot{\mathbf{x}}$, and $H.O.T.$ the higher order terms that can be neglected. By definition, the corresponding state derivative $\dot{\mathbf{x}}_0$ satisfies:

$$\dot{\mathbf{x}}_0 \equiv \mathbf{f}(\mathbf{x}_0) + \mathbf{G}(\mathbf{x}_0)\mathbf{u}_0 \tag{5}$$

Using this expression and the standard linear definition,

$$\mathbf{A}_0 = \frac{\partial}{\partial \mathbf{x}}\left[\mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}\right]\Big|_{\substack{\mathbf{x}=\mathbf{x}_0\\ \mathbf{u}=\mathbf{u}_0}} \tag{6a}$$

$$\mathbf{B}_0 = \frac{\partial}{\partial \mathbf{u}}\left[\mathbf{G}(\mathbf{x})\mathbf{u}\right]\Big|_{\substack{\mathbf{x}=\mathbf{x}_0\\ \mathbf{u}=\mathbf{u}_0}} = \mathbf{G}(\mathbf{x}_0) \tag{6b}$$

with $\mathbf{A}_0$ and $\mathbf{B}_0$ being the partials evaluated at the current reference point $[\mathbf{x}_0, \mathbf{u}_0]$ on the state/control trajectory; Equation (4), i.e., the approximation of $\dot{\mathbf{x}}$ for $\mathbf{x}$ and $\mathbf{u}$ in the neighborhood of $[\mathbf{x}_0, \mathbf{u}_0]$ can be written as:

$$\dot{\mathbf{x}} \cong \dot{\mathbf{x}}_0 + \mathbf{A}_0\,(\mathbf{x} - \mathbf{x}_0) + \mathbf{B}_0 \Delta\mathbf{u} \tag{7}$$

where $\Delta\mathbf{u} = (\mathbf{u} - \mathbf{u}_0)$ represents the incremental control command. This suggests that in a small neighborhood of the reference state we can approximate the nonlinear system (3) by its linearization about that reference state.

Considering this linear approximation in the second step of the backstepping procedure presented, we obtain the following control law for the increments of nonlinear control:

$$\Delta\mathbf{u} = \mathbf{G}^{-1}(\mathbf{x}_0)\left[-c_1\mathbf{z} - \dot{\mathbf{x}}_0 - \mathbf{A}_0\,(\mathbf{x} - \mathbf{x}_0) + \dot{\alpha} - \frac{\partial V_1(\xi)}{\partial \xi}\mathbf{k}(\xi)\right] \tag{8}$$

Moreover, considering small time increments and a sufficiently high control update rate, $\mathbf{x}$ approaches $\mathbf{x}_0$ much faster than an incremental change of the dynamics due to an incremental input, hence the incremental backstepping control law becomes:

$$\Delta\mathbf{u} = \mathbf{G}^{-1}(\mathbf{x}_0)\left[-c_1\mathbf{z} - \dot{\mathbf{x}}_0 + \dot{\alpha} - \frac{\partial V_1(\xi)}{\partial \xi}\mathbf{k}(\xi)\right] \tag{9}$$

This control ensures $\mathbf{z}$ to be uniformly ultimately bounded. Note that this control law results in increments of control commands; these changes must be added to the current reference command to obtain the full new control command input. Hence, the total control command is obtained as:

$$\mathbf{u} = \mathbf{u}_0 + \Delta\mathbf{u} \tag{10}$$

The incremental backstepping control law (10), as the application of backstepping to a system expressed in an incremental form, results in a control law that is not depending on the plant dynamics $\mathbf{f}(\mathbf{x})$ explicitly. This results in a *implicit*-control approach where the dependency of $\mathbf{f}(\mathbf{x})$ of the closed-loop system under feedback control is largely decreased, improving the system robustness against model mismatch and model uncertainties. Remaining dependency is due to changes in $\mathbf{f}(\mathbf{x})$ that are reflected in $\dot{\mathbf{x}}_0$, and since the control approach does require estimates of $\dot{\mathbf{x}}_0$ and $\mathbf{u}_0$, the control strategy is more sensor/actuator dependent. Moreover, apart from the aspects considered, the control needs as well the vehicle control derivatives $\mathbf{G}(\mathbf{x}_0)$. To make a clear difference with respect to standard (Jacobian) linearization over *operating* points, a graphical interpretation of the implicit nature of increments of control is depicted in Figure 3-(c). The incremental backstepping block diagram is illustrated in Figure 4.

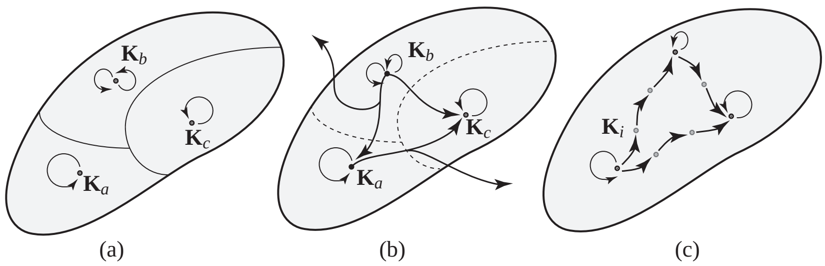

Fig. 3 Graphical interpretation of three control strategies: (a) some linear controllers designed over some operating points by standard (Jacobian) linearization of the system; (b) the concept of gain-scheduling between these operating points, where stability and convergence are not guaranteed overall; (c) the implicit nature of increments of control action, the current state represents a new reference and the control strategy acts stabilizing or tracking incrementally, and without the need of scheduling or the design of multiple controllers.

The implementation of incremental-based controllers considers the following assumptions:

(i) It is assumed to have complete and accurate knowledge about the state of the system. State derivatives (acceleration) sensors are considered to be available for this study as well. In the case of angular acceleration measurements, they may be measured directly or derived by diferentiation from inertial measurement unit (IMU) gyro measurements and filtered accordingly;

(ii) For small time increments, state derivatives evolve faster than the state upon fast control action, which directly influences the dynamics of the rigid body. In other words, the state only change by integrating state derivatives, hence making the difference $(\mathbf{x} - \mathbf{x}_0)$ negligible for for small time increments as compared to $\dot{\mathbf{x}}$;

(iii) Fast control action is assumed. This assumption complements the previous one in the sense that the dynamics of the actuators are considered to evolve much faster than the states. For this study a linear second order dynamics for the actuators is assumed, and considering an actuator undamped natural frequency ω_{n_c} sufficiently high guarantees the fast actuator requirement of incremental control action.

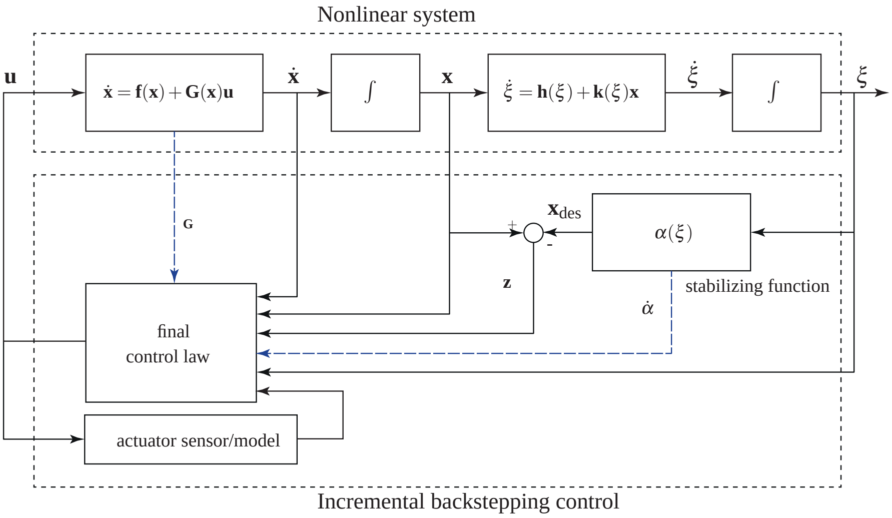

Fig. 4 Incremental backstepping control block diagram for second order cascaded systems. Dashed arrows represent information required for control design. Notice that the final control law in this case requires knowledge of $\mathbf{G}$, but also of $\dot{\mathbf{x}}$ and $\mathbf{u}_0$.

Regarding the actuator state requirement, Fig.5-(a) illustrates a sensor-dependency configuration, where the actuator state measurements are readily available (e.g. known current surface deflection), and Fig.5-(b) illustrates the model-dependent approach, where actuator state measurements are not readily available and a high-fidelity model of actuator dynamics are to be included in the control architecture as to supply the required control input reference $\mathbf{u}_0$. The mismatch of such measurements with respect to reality must be studied in order to avoid wind-up effects. Moreover, actuator state measurements may contain noise, biases, and delays. Of course, physical limitations exists and the attitude control system will depend on appropriate choice of sensors and actuators. In some particular cases, a combination of these two approaches may be necessary.

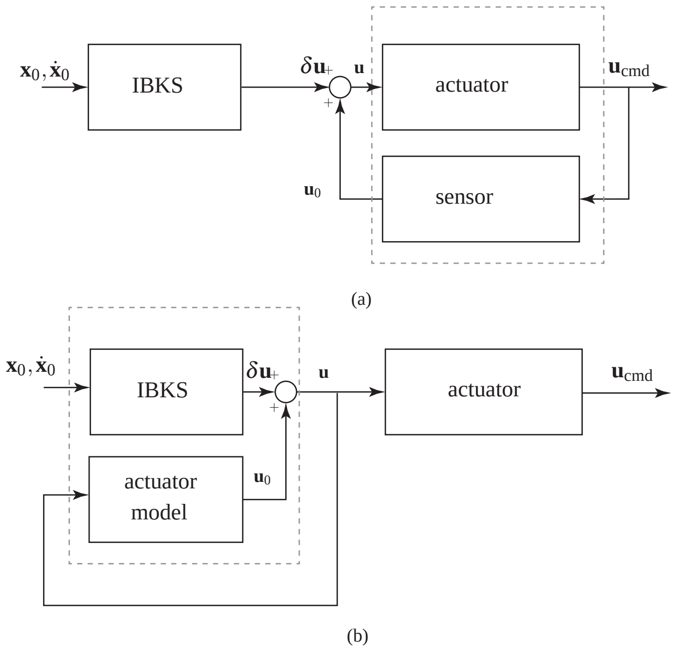

Fig. 5 Actuator state measurement/estimation architectures for incremental backstepping: (a) sensor-dependent. (b) model-dependent.

3 Flight Control Law Design

The incremental backstepping methodology has remained quite general up to this point. In the following, for flight control law design, we will demonstrate this concept considering attitude and rate control, outer and inner loop, respectively, by applying the methodology as a single-loop control for both systems simultaneously. Extra outer loops, see Fig. 6, could be also considered in such control law design with backstepping, but not shown here. Notice that in general, structures for flight control have at their core several blocks of dynamic inversion [7]. Such architectures are difficult to study from the stability point of view due to the multi-loop interconnection and time-scale separation, in contrast with backstepping-based de-

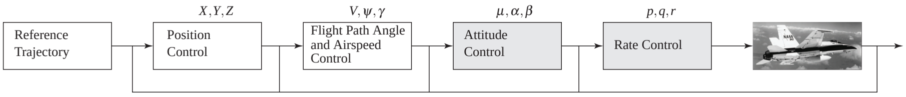

Fig. 6 Four loop feedback design for flight control. Grey boxes represent the attitude and rate control systems considered for flight control law design in the following. Image credits: [28].

sign which starts from the subsystem farthest from the control input and steps back through the integrators by considering augmented control Lyapunov functions (and hence from a stability view point) in a step-by-step fashion to obtain control laws for some desired motion with known stability and convergence properties.

In this sense, we demonstrate the incremental backstepping by considering Euler's equation of motion for the angular velocities of a vehicle in vector form:

$$\mathbf{M}_B = \mathbf{I}\dot{\omega} + \omega \times \mathbf{I}\omega \tag{11}$$

where $\omega \in \mathbf{R}^3$ is the angular velocity vector, $\mathbf{M}_B \in \mathbf{R}^3$ is the external (unknown) moment vector in body axes, and $\mathbf{I}$ the inertia matrix of the rigid body (with $x-z$ a plane of symmetry). We will be interested in the time history of the angular velocity vector, hence the dynamics of the rotational motion of a vehicle in Eq. (11) can be rewritten as the following set of differential equations:

$$\dot{\omega} = \mathbf{I}^{-1}\left(\mathbf{M}_B - \omega \times \mathbf{I}\omega \right) \tag{12}$$

where:

$$\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{bmatrix} \qquad \mathbf{M}_B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = SQ \begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix}$$

with p,q,r, the body roll, pitch, and yaw rates, respectively; L,M,N, the roll, pitch, and yaw moments, respectively; and S the wing surface area, Q the dynamic pressure, b the wing span, $\bar{c}$ the mean aerodynamic chord, and C_l, C_m, C_n the moment coefficients for roll, pitch, and yaw, respectively. Furthermore, let $\mathbf{M}_B$ be the sum of moments partially generated by the aerodynamics of the airframe (subscript a), moments generated by the control derivatives (subscript c) times the deflection of control surfaces (δ), and external disturbance moments (subscript d):

$$\mathbf{M}_B = \mathbf{M}_a + \mathbf{M}_c\delta + \mathbf{M}_d \tag{13}$$

where:

$$\mathbf{M}_a = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_a \qquad \mathbf{M}_c = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_c \qquad \delta = \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \qquad \mathbf{M}_d = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_d$$

and δ corresponds to the control inputs: aileron, elevator, and rudder deflection angles, respectively. Hence, the dynamic equation in consideration can be rewritten as:

$$\dot{\omega} = \mathbf{f}(\omega, \varsigma) + \mathbf{g}(\varsigma)\delta + \mathbf{d} \tag{14}$$

with:

$$\mathbf{f}(\omega, \varsigma) = \mathbf{I}^{-1}\left(\mathbf{M}_a - \omega \times \mathbf{I}\omega\right) \qquad \mathbf{g}(\varsigma) = \mathbf{I}^{-1}\mathbf{M}_c \qquad \mathbf{d} = \mathbf{I}^{-1}\mathbf{M}_d$$

and $\varsigma \in \mathbf{R}^p$ a parameter vector. For the rotational motion, this equation becomes:

$$\dot{\omega} = \mathbf{I}^{-1}\left(\mathbf{M}_a - \omega \times \mathbf{I}\omega\right) + \mathbf{I}^{-1}\mathbf{M}_c\delta + \mathbf{I}^{-1}\mathbf{M}_d \tag{15}$$

Without knowledge of the disturbances, and introducing the virtual control input $\nu = \dot{\omega}_{\text{des}}$, applying nonlinear dynamic inversion (NDI) to Eq. (15) results in an expression for the control input of the vehicle as:

$$\delta = \mathbf{M}_c^{-1}\left(\mathbf{I}\nu - \mathbf{M}_a + \omega \times \mathbf{I}\omega\right) \tag{16}$$

This resulting NDI control law depends on accurate (full) knowledge of the aerodynamic model contained in both $\mathbf{M}_a$ and $\mathbf{M}_c$, and hence depends on the model uncertainties contained therein. Furthermore it also depends on parametric uncertainties regarding inertia parameters, center of gravity, misalignment, etc. Such a dynamic inversion control law is intended to linearize and decouple the (inner loop) rotational dynamics in order to obtain an explicit desired closed loop dynamics to be followed. Notice that this result does not consider the effect of the external disturbance $\mathbf{d}$, and hence does not reject it properly. In the following, we are interested to go further using the result from backstepping for a more flexible and augmented design.

For the sake of simplicity, we will depart the study from *Step 2* of the backstepping design procedure explained before, assuming that outer-subsystem's stabilizing control laws are already obtained and stepped back up to the dynamic equation in consideration. In this sense, we depart from the final error-dynamics equation:

$$\dot{\mathbf{z}} = \dot{\omega} - \dot{\alpha}(\sigma, \omega) = \mathbf{f}(\omega, \varsigma) + \mathbf{g}(\varsigma)\delta - \dot{\alpha}(\sigma, \omega) \tag{17}$$

where σ may represent a kinematic variable or a state stepped back from the outer-subsystems. For flight control law design, the goal is to stabilize the complete system described by the following augmented equation:

$$\dot{\mathbf{z}} = \mathbf{I}^{-1}\left(\mathbf{M}_a - \omega \times \mathbf{I}\omega\right) + \mathbf{I}^{-1}\mathbf{M}_c\delta + \mathbf{I}^{-1}\mathbf{M}_d - \dot{\alpha}(\sigma, \omega) \tag{18}$$

and with partial knowledge of the disturbance (full knowledge is practically impossible), and applying backstepping to Eq. (18) in combination with a nonlinear damping term Γ_d [17, 18, 29] to handle the disturbance effect and control input uncertainty, a plaussible expression for the control input of the vehicle results in:

$$\delta = \mathbf{M}_c^{-1}\mathbf{I}\Big[-\mathbf{K}_\omega\mathbf{z} - \mathbf{I}^{-1}\big(\mathbf{M}_a - \omega \times \mathbf{I}\omega\big) + \dot{\alpha}(\sigma,\omega) + \Gamma_d\Big] \tag{19}$$

with $\mathbf{K}_\omega > 0_{3\times3}$. This control ensures $\mathbf{z}$ to be uniformly ultimately bounded, meaning that the complete system is stabilized, and the flexibility of the method allows to consider several families of control laws apart from a pure linearizing one. Moreover, the flexibility due to CLF augmentation and redesign allowes the inclusion of a nonlinear damping term Γ_d to reject external disturbance effect and possible input uncertainty. Again, the resulting control law depends on accurate (full) knowledge of the aerodynamic model contained in both $\mathbf{M}_a$ and $\mathbf{M}_c$, and hence also depends on the model uncertainties contained therein. For this reason, we complete the study by improving the robustness of such backstepping design by introducing its incremental counterpart, using the implicit approach with the recursive control law:

$$\delta = \delta_0 + \mathbf{M}_c^{-1}\mathbf{I}\Big[-\mathbf{K}_\omega\mathbf{z} - \dot{\omega}_0 + \dot{\alpha}(\sigma,\omega) + \Gamma_d\Big] \tag{20}$$

Which results in a stabilizing control law for outer-loop variables that is not depending on the aerodynamic model $\mathbf{M}_a$, hence it will not be affected by its uncertainties. In this case, the aerodynamic (control input) uncertainty present in $\mathbf{M}_c$, the parametric uncertainty, and the effect of external disturbance, are captured by the vehicle's accelerations and by the implicit architecture of the closed-loop system. Moreover, the extra nonlinear damping term may be suitable to alleviate this problem even further, but its contribution to the closed-loop robustness is not studied here.

4 Robustness

Apart from the robustness properties already discussed before, the present section shows briefly closed-loop forms of the systems in consideration under feedback control for particular uncertainty structures. Ignoring the external disturbance for this analysis (and hence the nonlinear damping term), the application of the backstepping control law in Eq. (19) on the nominal system (18) results in the following stable closed-loop error-dynamics:

$$\dot{\mathbf{z}} = -\mathbf{K}_\omega\mathbf{z} \tag{21}$$

Instead, if we consider the uncertain system with the fact that the error-dynamics (17) may contain uncertainties from the original dynamics as, for instance:

$$\dot{\mathbf{z}} = \mathbf{f}(\omega,\varsigma) + \Delta\mathbf{f}(\omega,\varsigma) + \big[\mathbf{g}(\varsigma) + \Delta\mathbf{g}(\varsigma)\big]\delta - \dot{\alpha}(\sigma,\omega) \tag{22}$$

the application of the backstepping control law in Eq. (19) does not robustify the closed-loop dynamics against model and parametric uncertainty present in both $\Delta\mathbf{f}(\omega,\varsigma)$ and $\Delta\mathbf{g}(\varsigma)$, besides from the aerodynamic uncertainty contained therein,

$$\dot{\mathbf{z}} = -\left[\mathbf{I} + \frac{\Delta\mathbf{g}(\varsigma)}{\mathbf{g}(\varsigma)}\right]\mathbf{K}_\omega\mathbf{z} + \Delta\mathbf{f}(\omega,\varsigma) - \frac{\Delta\mathbf{g}(\varsigma)}{\mathbf{g}(\varsigma)}\left[\mathbf{f}(\omega,\varsigma) + \dot{\alpha}(\sigma,\omega)\right] \tag{23}$$

unless considering the robustification with a better nonlinear damping design or via robust backstepping, which will make the control law more conservative, see [18, 29].

As a matter of fact, we are interested on robustness properties from incremental backstepping. For the *partly*-linearized nonlinear system, recall we assume in this case angular accelerations to be known accurately, hence $\mathbf{f}(\omega,\varsigma)$ represents $\dot{\omega}_0$ and not $\mathbf{I}^{-1}\left(\mathbf{M}_a - \omega \times \mathbf{I}\omega\right)$. Such difference is important since it not only represents a measurement versus an explicit model containing aerodynamic terms and parameters, but also because the term $\Delta\mathbf{f}(\omega,\varsigma)$ is no longer present in such case since such measurement uncertainty is considered negligible. For this reason, the uncertain system is rewritten as:

$$\dot{\mathbf{z}} = \dot{\omega}_0 + \left[\mathbf{g}(\varsigma) + \Delta\mathbf{g}(\varsigma)\right]\Delta\delta - \dot{\alpha}(\sigma,\omega) \tag{24}$$

and applying the incremental backstepping control law to such uncertain system results in:

$$\dot{\mathbf{z}} = -\left[\mathbf{I} + \frac{\Delta\mathbf{g}(\varsigma)}{\mathbf{g}(\varsigma)}\right]\mathbf{K}_\omega\mathbf{z} - \frac{\Delta\mathbf{g}(\varsigma)}{\mathbf{g}(\varsigma)}\left[\dot{\omega}_0 + \dot{\alpha}(\sigma,\omega)\right] \tag{25}$$

which only contains uncertainties in the control derivatives and moments of inertia.

5 Example: longitudinal missile control

In this section the advantage of incremental backstepping is demonstrated with an example consisting on the tracking control design for a longitudinal missile model. This example is adapted from [28]. A second order nonlinear model of a generic surface-to-air missile as obtained from [16] is considered. The model consists of the longitudinal force and moment equations representative of a missile traveling at an altitude of approximately 6000 meters, with aerodynamic coefficients represented as third order polynomials in angle of attack α and Mach number M.

The nonlinear equations of motion in the pitch plane are given by

$$\dot{\alpha} = q + \frac{\bar{q}S}{mV_T}\left[C_z(\alpha,M) + b_z(M)\delta\right] \tag{26a}$$

$$\dot{q} = \frac{\bar{q}Sd}{I_{yy}}\left[C_m(\alpha,M) + b_m(M)\delta\right] \tag{26b}$$

where:

$$C_z(\alpha,M) = \varphi_{z1}(\alpha) + \varphi_{z2}(\alpha)M \qquad b_z(M) = 1.6238M - 6.7240$$
$$C_m(\alpha,M) = \varphi_{m1}(\alpha) + \varphi_{m2}(\alpha)M \qquad b_m(M) = 12.0393M - 48.2246$$

and:

$$\varphi_{z1}(\alpha) = -288.7\alpha^3 + 50.32\alpha|\alpha| - 23.89\alpha \quad \varphi_{z2}(\alpha) = -13.53\alpha|\alpha| + 4.185\alpha$$
$$\varphi_{m1}(\alpha) = 303.1\alpha^3 - 246.3\alpha|\alpha| - 37.56\alpha \quad \varphi_{m2}(\alpha) = 71.51\alpha|\alpha| + 10.01\alpha$$

These approximations are valid for the flight envelope $-10^\circ \leq \alpha \leq 10^\circ$ and $1.8 \leq M \leq 2.6$. To facilitate the control design, the nonlinear missile model is rewritten in the more general state-space form as:

$$\dot{x}_1 = x_2 + f_1(x_1) + g_1 u \tag{27a}$$
$$\dot{x}_2 = f_2(x_1) + g_2 u \tag{27b}$$

where:

$$x_1 = \alpha \qquad x_2 = q$$
$$f_1(x_1) = C_1\left[\varphi_{z1}(x_1) + \varphi_{z2}(x_1)M\right] \qquad f_2(x_1) = C_2\left[\varphi_{m1}(x_1) + \varphi_{m2}(x_1)M\right]$$
$$g_1 = C_1 b_z \qquad g_2 = C_2 b_m$$
$$C_1 = \frac{\bar{q}S}{mV_T} \qquad C_2 = \frac{\bar{q}Sd}{I_{yy}}$$

The control objective considered here is to design an autopilot with the incremental backstepping method that tracks a command reference y_r (all derivatives known and bounded) with the angle of attack x_1. It is assummed that the aerodynamic force and moment functions are *not* exactly known and the Mach number M is treated as a parameter available for measurement. Furthermore, the contribution of the fin deflection on the right-hand side of the force equation (27a) is ignored during the control design, since the backstepping method can only handle nonlinear systems of lower-triangular form, i.e. the assumption is made that the fin surface is a pure moment generator. This is a valid assumption for most types of aircraft and aerodynamically controlled missiles, often made in flight control systems design [28].

We begin the control design procedure with standard backstepping for illustration purposes and further comparisons.

Step 1: First, introduce the tracking errors as:

$$z_1 = x_1 - y_r \tag{28a}$$
$$z_2 = x_2 - \alpha_1 \tag{28b}$$

where α_1 is the stabilizing function to be designed as a first design step (and not to be confused with α, the angle of attack). The z_1-dynamics satisfy:

$$\dot{z}_1 = x_2 + f_1 - \dot{y}_r = z_2 + \alpha_1 + f_1 - \dot{y}_r \tag{29}$$

Consider a candidate CLF V_1 for the z_1-subsystem defined as:

$$V_1(z_1) = \frac{1}{2}\left(z_1^2 + k_1\lambda_1^2\right) \tag{30}$$

where the gain $k_1 > 0$ and the integrator term $\lambda_1 = \int_0^t z_1 \mathrm{d}t$ are introduced to robustify the control design against the effect of the neglected control term. The derivative of V_1 along the solutions of (29) is given by:

$$\dot{V}_1 = z_1\dot{z}_1 + k_1\lambda z_1 = z_1\left(z_2 + \alpha_1 + f_1 - \dot{y}_r + k_1\lambda_1\right) \tag{31}$$

The stabilizing function α_1 is selected as:

$$\alpha_1 = -c_1z_1 - k_1\lambda_1 - f_1 + \dot{y}_r, \quad c_1 > 0 \tag{32}$$

to render the derivative

$$\dot{V}_1 = -c_1z_1^2 + z_1z_2 \tag{33}$$

The cross term z_1z_2 will be dealt with in the second design step.

Step 2: Second, the z_2-dynamics are given by:

$$\dot{z}_2 = f_2 + g_2u - \dot{\alpha}_1 \tag{34}$$

where $\dot{\alpha}_1 = -c_1(x_2 + f_1 - \dot{y}_r) - k_1z_1 - \dot{f}_1 + \ddot{y}_r$. The CLF V_1 is augmented with an additional term to penalize z_2:

$$V_2(z_1, z_2) = V_1 + \frac{1}{2}z_2^2 \tag{35}$$

The derivative of V_2 along the solutions of (29) and (34) satisfies

$$\dot{V}_2 = -c_1z_1^2 + z_1z_2 + z_2\big(f_2 + g_2u - \dot{\alpha}_1\big) = -c_1z_1^2 + z_2\big(z_1 + f_2 + g_2u - \dot{\alpha}_1\big) \tag{36}$$

Notice that the first term in the right-hand of the last expression is already negative semi-definite. Hence, a control law for u can now be defined to cancel all indefinite terms, and the most straightforward choice is given by:

$$u = \frac{1}{g_2}\big(-c_2z_2 - z_1 - f_2 + \dot{\alpha}_1\big) \tag{37}$$

According to the results previously outlined, the incremental backstepping control law design follows from considering the approximate dynamics around the current reference state for the dynamic equation of the pitch rate:

$$\dot{q} \cong \dot{q}_0 + \frac{\bar{q}Sd}{I_{yy}}b_m(M)\Delta\delta \tag{38}$$

assuming that pitch acceleration is available for measurement, and which is rewriten in our formulation as:

$$\dot{x}_2 \cong \dot{x}_{2_0} + g_2\Delta u \tag{39}$$

From there, the design procedure is the same as before. It suffices to consider the new $f_2 = \dot{x}_{2_0}$, noticing that we are replacing the accurate knowledge of f_2 by a measurement (or an estimate) instead, and this trade-off results in a robustified backstepping control law which is not entirely dependent on a model.

The incremental backstepping control law is hence obtained as:

$$u = u_0 + \frac{1}{g_2}\left(-c_2 z_2 - z_1 - \dot{x}_{2_0} + \dot{\alpha}_1\right) \tag{40}$$

Simulation results for the backstepping controller in Eq. (37) and the incremental backstepping controller in Eq. (40) are now presented. The maneuver simply consists on a smooth doublet angle-of-attack trajectory for the missile. Figure 7 shows the tracking control numerical simulation at Mach 2.0 of the nominal (idealized) longitudinal missile model for the two control laws derived at the same gain selections of $k_1 = c_1 = c_2 = 10$, showing relatively the same performance and closed-loop response as expected with no uncertainty and model mismatch.

Now we introduce aerodynamic uncertainties modeled as real parametric uncertainty of the coefficients present in C_z, b_z, C_m, b_m. The coefficients are perturbed from their nominal value within a $\pm 20\%$ range. Figure 8 shows tracking control numerical simulation of the uncertain longitudinal missile model for the backstepping controller in Eq. (37) and with the same gain selection. As expected, this conventional backstepping alone is robust but not quite much over large dynamic uncertainties, and hence the nominal performance is lost and/or degraded.

For this particular example, the tracking capability and superior robustness at Mach 2.0 of the uncertain longitudinal missile model are verified, showing a great benefit of the incremental version over conventional backstepping designs since the new structure is able to cope very well with relatively large aerodynamic uncertainty, and hence the nominal performance is not lost and/or degraded significantly.

6 Conclusion

This paper presented a robust nonlinear flight control strategy based on results combining incremental control action with the backstepping design methodology, called incremental backstepping. Such approach is aimed to enhance the robustness of flight control systems in the presence of large model and parametric uncertainties.

The incremental feature enhances robustness capabilities by reducing feedback control dependency on accurate knowledge of the baseline aircraft model, where only information on control derivatives is required. Changes in aerodynamics causes forces and moments which affect the vehicle dynamics, which in turn may be captured or measured by accelerometers. Hence, vehicle's sensitivity to its baseline model is reduced in favor of obtaining a robust measure of vehicle's acceleration.

The use of this type of control action, which requires information of actuator states and accelerations, make these sensor-based type of controllers efficient in terms of performance, and robust in terms of handling uncertainties. Unlike con-

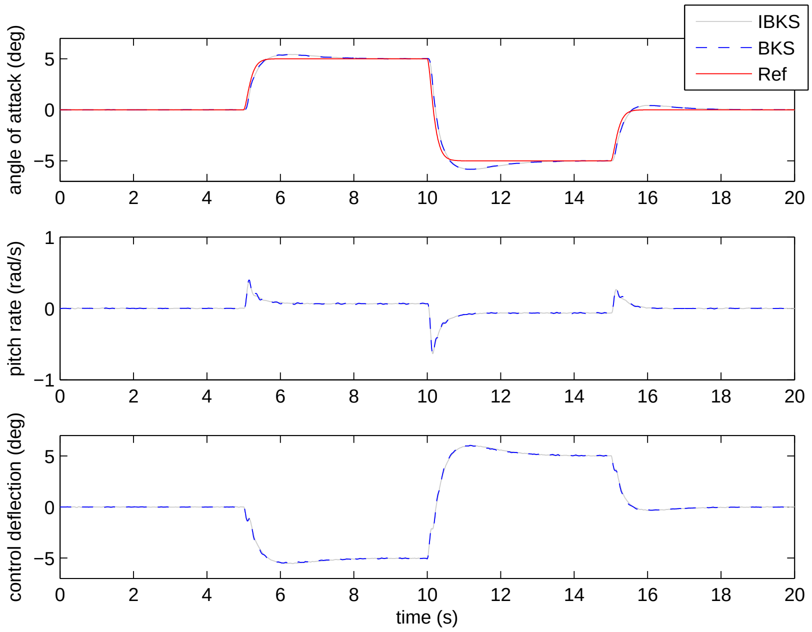

Fig. 7 Backstepping (37) and incremental backstepping (40) tracking control numerical simulation of the nominal longitudinal missile model for a gain selection of $k_1 = c_1 = c_2 = 10$.

ventional backstepping, this control design technique is implicit in the sense that desired closed-loop dynamics do not reside in some explicit model to be cancelled but result when the feedback loops are closed.

The potential of incremental backstepping was evidenced in the context of an example for the longitudinal tracking of a conventional missile model, which showed that performance was not severely degraded upon relatively large variations in the missile aerodynamic model.

In practice, however, incremental backstepping-based control rely on accurate actuator state and acceleration measurements which may not be readily available or which may contain noise, biases, and delays, hence a disadvantage of this type of architectures which may be further studied on.

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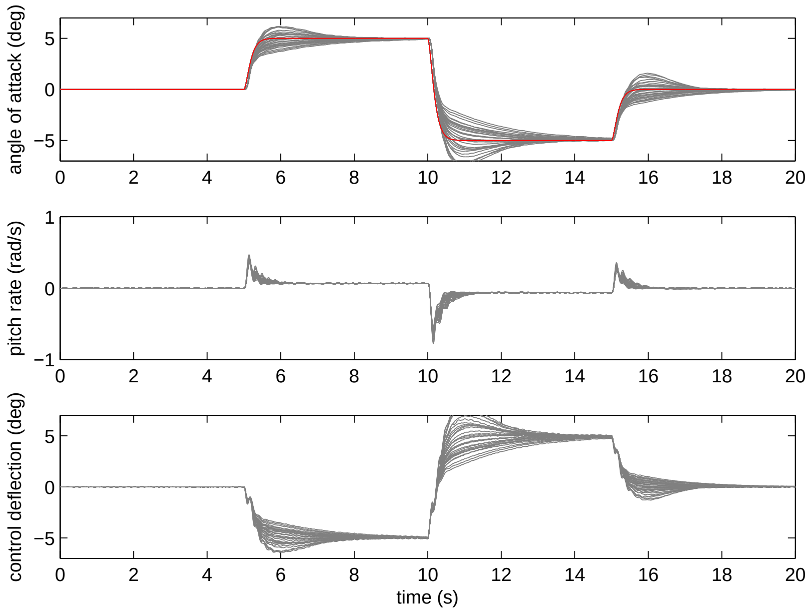

Fig. 8 Backstepping (37) tracking control numerical simulation of the uncertain longitudinal missile model for a gain selection of $k_1 = c_1 = c_2 = 10$. Aerodynamic uncertainties are modeled as real parametric uncertainty of the coefficients present in C_z, b_z, C_m, b_m. The coefficients are perturbed from their nominal value within a $\pm 20\%$ range.

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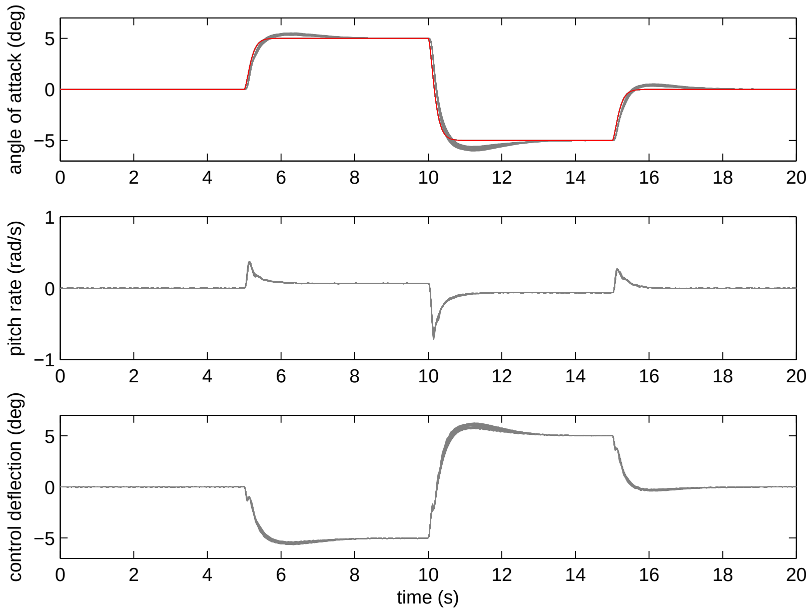

Fig. 9 Incremental backstepping (40) tracking control numerical simulation of the uncertain longitudinal missile model for a gain selection of $k_1 = c_1 = c_2 = 10$. Aerodynamic uncertainties are modeled as real parametric uncertainty of the coefficients present in C_z, b_z, C_m, b_m. The coefficients are perturbed from their nominal value within a $\pm 20\%$ range.

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