Proceedings of the EuroGNC 2013, 2nd CEAS Specialist Conference
on Guidance, Navigation & Control, Delft University of Technology,
Delft, The Netherlands, April 10-12, 2013

On the Crucial Role of the Estimation in Interception Endgames.

Josef Shinar[1]
Technion-Israel Institute of Technology
Haifa 32000, Israel

Vladimir Turetsky[2]
Ort Braude College, Karmiel 21982, Israel
Extended abstract

Abstract. The paper discusses the crucial function of the estimation process in a realistic (measurement noise corrupted) interception endgame and indicates how to improve the homing performance by a suitable estimator design. The different error sources, due to the measurement noise and the need of using an estimator in the guidance loop are described and a new approach for reducing the effects of the estimation error is proposed. A criterion for suitable endgame estimator is formulated, followed by an innovative estimator design approach, based on using the available time-to-go information.

1 Introduction

The homing guidance of an interceptor missile is a stochastic optimal control problem with the objective to minimize, using a sequence of noise corrupted measurements, the expected value of the miss distance. For using these measurements as a basis of a feedback control, the noisy signals have to be filtered. Eventual target maneuvers are considered as unknown bounded disturbances. Classical guidance laws, like Proportional Navigation did not required exact knowledge of the target maneuvers. Satisfactory homing performance, i. e. small miss distances, could be achieved by a rather large maneuverability advantage (more than 3) over the target.

In modern guidance laws the knowledge of the actual target acceleration is necessary for achieving with a reasonable modest maneuverability advantage sufficiently small miss distances. Unfortunately, the acceleration of another object cannot be measured from a moving platform. In the ideal case, where all measurements are noise free and the dynamic model is perfectly known, the unmeasured variable can be reconstructed by an observer. In reality, such reconstruction has to be made, using the available noise corrupted measurements, by an estimator. Thus, in an interceptor guidance system the estimator is an indispensable element, performing a dual role, the role of a filter and the role of an observer. The homing performance of an interceptor missile is limited by the estimation accuracy.

The objective of this paper is to present the different error sources created by the estimation process in an interception endgame and to propose an innovative estimator design approach for improving the homing performance.

[1] 1Professor Emeritus, Faculty of Aerospace Engineering, aer4301@aerodyne.technion.ac.il
[2] Associate Professor, Department of Mathematics, turetsky1@braude.ac.il

2 Estimation Error Components

The estimation error consists of two components. The first one is dynamic in nature and expressed by the delay occurring during the convergence of the estimated state variables. The second component is of stochastic nature and expressed by the variance of the converged estimate. Due to the different nature of the error components no clear unique definition of an *optimal* estimator can be found in the literature.

The design of a Kalman filter [1] minimizes the second error component. This approach has been adopted, because most control processes are of long duration and abrupt variations of the state variables are not expected. In such cases the estimation delay is not critical.

In an interception endgame the situation is different. Such an endgame is of short duration and estimation errors occurring near the final time are crucial. Therefore, the estimation process has to become faster as the final time of the endgame is approaching. Within the modern guidance laws the knowledge of the time-togo is an essential element and a considerable effort is invested to obtain it accurately. However, the currently used estimation processes completely ignore this monotonically changing variable, although it is available in the guidance system. For effective terminal guidance this important information has to be included in the estimation process.

In an interception endgame against maneuvering targets the unknown target acceleration is a set of random inputs. Such inputs can be considered as a stochastic process and approximated by the output of a (linear) *shaping filter* driven by Gaussian white noise [2]. It is well known that for linear systems with zero-mean, white and Gaussian measurement and process noises the Kalman filter, based on the correct model of the system dynamics, is the optimal minimum variance estimator. The measurement noise used in interception simulations has generally such characteristics, but the representation of a random target maneuver as the output of a shaping filter driven by a zero-mean, white, Gaussian noise is only an approximation. Moreover, the random acceleration commands can be discontinuous, representing a random jump process. They are bounded, but certainly neither white, nor Gaussian. In several recent papers [3, 4] it was shown that in such cases the optimal estimator is of infinite dimension. Thus, every computationally feasible (finite dimensional) estimator can be, at best, only suboptimal.

In order to satisfy the requirement for a small miss distance in interception endgames, the estimation process has to minimize both the estimation delay and the variance of the converged estimation error. These requirements are contradictory. The convergence time required to identify a rapid input change is composed of the detection time and the estimator's response time. Short detection time comes at the price of high false alarm rate, while short response time requires wide bandwidth, generating large estimation errors. Good filtering, providing a small estimation error variance, requires narrow bandwidth leading to a slow response.

Detecting the variation in a state variable (for example target acceleration) requires that the lateral separation due to this variation will be larger than measurement uncertainty. Therefore, the detection time depends on the noise characteristics and the range from the target. If the measurement noise is of a constant angular value, as in most electro-optic seekers, the detection time decreases with the range. The estimator's response time depends on its bandwidth, which is constant in most conventional estimator designs. The response time can be reduced by increasing the bandwidth at the expense of less efficient filtering, which leads to larger residual estimation errors and increased miss distances for target acceleration variations occurring in the early part of the endgame. Due to the contradictory nature of the estimation error components, an acceptable satisfactory compromise by a single estimator has not yet been found.

3 Separation and Certainty Equivalence in Realistic Interceptions

In principle, a stochastic optimal control problem is solved by stochastic dynamic programming. This is, however, a rather complex approach even for relatively simple problems. Fortunately, the interception endgame guidance can be in most cases linearized. In order to reduce the complexity of solving stochastic optimal control problems for linear systems, two important properties were asserted long time ago.

The first one is called the *separation* property [5] and it says that control and the estimation logic can be derived and optimized separately. Another closely related (but not identical) property is the *certainty equivalence* , which states that the optimal control function of a stochastic optimal control problem is the same as the one of the associated *deterministic* optimal control problem. The only difference is that the (unknown) state variables are replaced by their estimated values.

The validity of the *certainty equivalence* property has been rigorously proven long time ago [5] for linear quadratic problems with white Gaussian noise and was extended later to include also the cases with non-Gaussian and colored noise [6] as well as non quadratic cost [7].

Realistic interceptor guidance problems are characterized, in addition to noise corrupted measurements, by bounded controls, and saturated state variables and non-Gaussian random disturbances. The validity of the *separation* and *certainty equivalence* properties has never been proven for this class of problems [8].

In spite of that, in the 50 years long history of guided missiles it has been of common practice to design the estimators and missile guidance laws independently. In most cases such comfortable design approach has been acceptable, because it succeeded to satisfy the performance requirements, due to the substantial maneuverability advantage of guided missiles over their manned aircraft targets. In a more stressing scenario, such as Ballistic Missile Defense, the usefulness of relying on the *separation* and *certainty equivalence* properties becomes strongly questionable.

For cases where the *certainty equivalence* property cannot be proven, a *"partial" separation* property was asserted [9], stating that the estimator can be designed independently of the controller, but the derivation of the optimal control function has to be based on the conditional probability density function (conditioned on the measurement history) of the estimated state variables. Unfortunately, this interesting approach provides no direction what is an *optimal* estimator for a well defined control (or guidance) task.

A few recent papers [10-11] adopted this approach and proposed different ways to optimize interceptor guidance laws based on the results of an independently designed estimation process. The problem of this promising new approach is the heavy computational demand requiring, at least presently, off-line computations. Therefore, at the present such guidance laws cannot be directly implemented in real time on board a homing interceptor missile.

4 Estimation in Three-Dimensional Space

In most of the guidance studies the analysis was limited to the planar case, mainly for sake of simplicity. Although the interception of a straight flying target takes place in the so called *collision* plane, the eventual target acceleration is generally out of this plane. Therefore, the interception of a maneuvering target has to be analyzed in three-dimensional space.

Assuming that the interception endgame starts near to the *collision course,* the last phase of the homing takes place in the direction of the target acceleration vector. This direction is unknown by the interceptor missile and has to be determined by the estimation process. The estimation is performed in the coordinate frame of the missile's seeker by two planar estimators perpendicular each other, yielding two components of the target acceleration vector normal to the line of sight. Even if the two planar estimators are identical, it cannot be guaranteed that the convergence of the two target acceleration components, which are in general of different magnitude, will be simultaneous. This phenomenon creates an additional error source in the homing process by increasing the time until the estimated target maneuver becomes useful for terminal guidance.

5 Diagnosis of Planar Endgames

Simulations of a planar interception end-game scenario [12] demonstrated the problem caused by the noise corrupted measurements requiring the presence of an estimator. The estimator in these simulations was a Kalman filter augmented with a *shaping filter* using an exponentially correlated acceleration (ECA) model [13]. Such a shaping filter has first order dynamics with two tuning parameters, the correlation time of the maneuver τ_s and the (assumed) level of the *process noise*, expressed by its standard deviation $\sigma_s = a_E^{max}/C_s$. The simulations used a differential game based guidance law (denoted DGL/1) [14] and a set of parameters that guarantee, in the ideal case of perfect information and without an estimator in the guidance loop, zero miss distances against all admissible bounded target maneuvers. However, the simulation results for a large set of interceptions, where the target performs in a short duration (4 sec) interception endgame randomly changing *bang-bang* type evasive maneuvers, demonstrate a very different outcome.

On Fig. 1 the homing performance, expressed by the average miss distance of a large number of Monte Carlo simulations is depicted as the function of the time-to-go for the direction change of the maneuver (time-to-go$_{sw}$). Moreover, this figure shows that the interception endgame is divided into two regions of different homing performance by a c*ritical* time-to-go, $(t_{go})_{cr}$ serving as the boundary between the regions of small and large miss distances. This *critical* time-to-go is composed of the *delay* due to the convergence of the estimated state variables (including also the maneuver detection) and the reaction time of the interceptor autopilot. For large values of time-to-go ({ t_{go}) > $(t_{go})_{cr}$) the average miss distance is rather small and is almost constant.

These simulation results demonstrate that small miss distances can be achieved only if the direction change of the target acceleration starts in the early part of the endgame (before the *critical* time-to-go). In this case, sufficient time remains until the closest approach for the estimated acceleration to converge to its true quasisteady value and the correction needed to acceptable interception. The Kalman filter design minimizes the variance of the converged estimation error and the guidance law receives soon enough almost correct values of the *zero-effort miss distance* for achieving good homing precision.

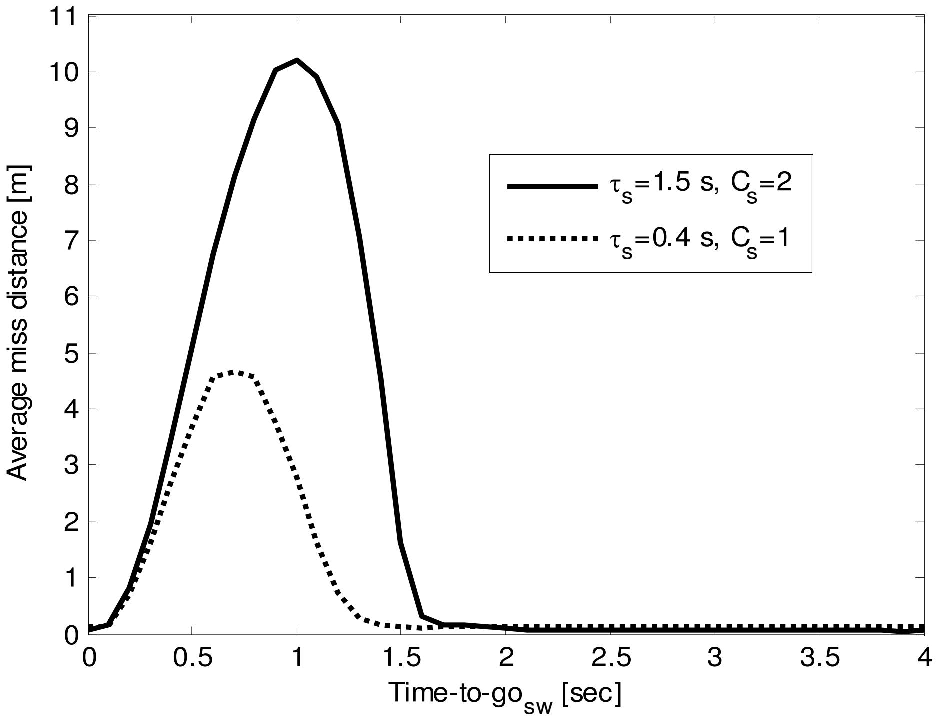

Fig. 1 Homing performance of DGL/1 against randomly switched "bang-bang" maneuver (full line-slow estimator; dotted line- fast estimator)

If the target acceleration starts later, the combination of the same estimator with the same guidance law fails to provide satisfactory results because of the estimation delay. By widening the bandwidth of the estimator (see the dotted line in Fig. 1) the *critical* time-to-go and the maximum miss distance are reduced at the expense of enlarged miss distances against maneuvers that change the direction in the earlier part of the endgame.

6 Delay Compensating Guidance Laws

Since the design of the Kalman filter provides minimum variance, some investigations concentrated to compensate the effect of the estimation delay. One approach in this direction is to include the estimation delay in a deterministic model of the interception problem by neglecting the stochastic features of the estimation process. Based on such approximation the interception scenario of a maneuvering target is reformulated as a delayed information pursuit-evasion game with bounded controls [15, 16]. In this formulation the evader has *perfect information* on all the state variables, as well as on the estimation delay of the pursuer.

The game solution is based on an intuitive approach, inspired by the idea of *reachable sets*. At every point of the time the *reachable set* of the evader is based on the information available to the pursuer. The objective is to reach the center of the convex hull of this *reachable set*. The interceptor guidance law (denoted DGL/C) is determined by the optimal pursuer strategy of this game. Each estimation delay model led to formulate a different deterministic pursuit-evasion game for the interception end-game of a maneuverable target.

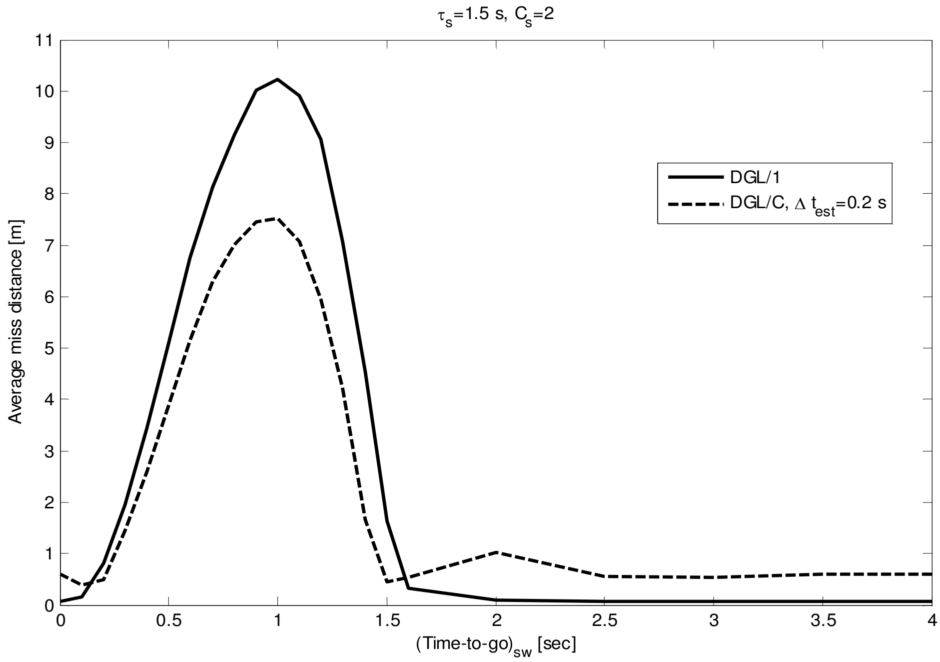

Fig. 2 Homing performance of DGL/1 and DGL/C against randomly switched "bang-bang" command with a slow estimator (τ_s=1.5 s, C_s=2).

The optimality of the game solution from the interceptor missile point of view is the reduction of the guaranteed (worst case) miss distance compared to the original (non compensated) guidance law. This reduction came in each case at the expense of increased miss distances against non optimal target maneuvers, as it can be seen in Fig. 2. For this reason the practical value of such guidance laws is not obvious.

Nevertheless, these guidance laws provide evidence that the *certainty equivalence* property is not valid in the case of interceptor guidance with noise corrupted measurements.

7 Integrated Estimation and Guidance

Not being able to rely on the questionable *separation* and *certainty equivalence* properties for stressing scenarios (e.g. in Ballistic Missile Defense, an *integrated* estimation and guidance algorithm was proposed and presented in detail [12]. This algorithm is based on an engineering approach using the insight gained by the understanding the inherent limitations of the classical estimation in short duration interception endgames. The *integrated* estimation and guidance algorithm in [12] applied different estimation strategies before and after the *critical* time-to-go and also included some modifications in the original (perfect information) guidance law DGL/1. This "logic based" *integrated* algorithm, developed for a planar scenario, showed encouraging results and was validated also in three dimensional interception endgames [17].

In spite of this success, in order to provide solution for effective and robust interceptor guidance against randomly maneuvering targets in realistic, noise corrupted scenarios, further investigations are still needed to develop a more general estimation/guidance algorithm. Since the robustness of the differential game based guidance law has been already demonstrated [2], the new directions consider mainly the estimation aspect, such as establishing a criterion for a suitable estimator performance for short range interception endgames against randomly maneuvering targets and the development of an innovative estimator design that explicitly considers the time-to-go in the estimation process of randomly maneuvering target acceleration.

8 Guidelines for Suitable Estimator Design in Interception Endgames

Based on the insight gained from [15] the criterion of *suitable* estimator performance for short range interception endgames against randomly maneuvering targets can be formulated in the following form: *An estimator is suitable for short range interception endgames against randomly maneuvering targets if its convergence time is shorter than the actual time-to-go minus the delay due to the autopilot dynamics.*

Based on this suitability criterion a new approach for estimator design can be developed. This approach is based on two elements:

1, The value of the *critical* time-to-go has to be reduced, subject to keeping on an acceptable level.the ensemble of the miss distances in the region of large time-to-go values ($(t_{go}) > (t_{go})_{cr}$

2, For the region of short values of time-to-go ($(t_{go}) < (t_{go})_{cr}$) a small number (as few as possible) *tuned* estimators have to be designed in order to guarantee a robust acceptable level of miss distances over the entire range.

In a *tuned* estimator the estimation delay is eliminated by correctly guessing the timing of the disturbance, such as the direction change of the *bang-bang* maneuver and including in the estimator model. Generally, such estimator also tolerates small discrepancies.

For very short values of the time-to-go terminal guidance law modifications may also be necessary, as in [12]. Naturally, such design is very much dependent on the relevant scenario parameters.

8 Conclusions

This paper presented the crucial role of the estimator in interception endgames. The main error sources in measurement noise corrupted interception endgame scenarios due to the need of including an estimator in the guidance loop were overviewed, Recent attempts for finding a satisfactory solution for effective guidance in realistic interception endgames against randomly maneuvering targets were also described.

There are several directions for further investigations that are needed to develop an improved (most probably, *integrated*) estimation/guidance algorithm to provide a solution for effective interceptor guidance against randomly maneuvering targets in general realistic (noise corrupted) scenarios. These directions are:

1, An innovative estimator design that explicitly considers the time-to-go in the estimation process of randomly maneuvering target acceleration, adapted to the interception scenario, as indicated briefly in this paper. .

2, Development of a theoretical and computationally feasible basis for creating an insight on the contribution of the estimator structure and its parameters to the probability distribution of the zero-effort miss distance and its propagation during the interception endgame in the presence of measurement noise.

In order to carry out successfully such investigations, close cooperation between the scientific communities dealing with estimation and terminal guidance is needed.

References:

1. Kalman, R.E. “A new approach of linear filtering and prediction problems”, *Trans. ASME,* Vol. 82D, 1960, 35.
2. Zarchan, P. “Representation of realistic evasive maneuvers by the use of shaping filters“, *J. Guidance, Contr.,* Vol. 2, No.1, 1979, 290.
3. Rotstein, H. and Szneier, M., “An exact solution to the general 4-blocks discretetime mixed H_2/H problems via convex optimization”, *IEEE Trans. Aut. Contr.,* Vol. AC-43, No. 6, 1998, 1475.
4. Lai, T. L. and Shan, Z., “Efficient recursive algorithms for detection of abrupt signal and systems”, *IEEE Trans. Aut. Contr.,* Vol. AC-44, No. 5, 1999, 952.
5, Joseph, P. D. and Tau, J. T. “On linear control theory” *Trans. AIEE*, Part III, Vol. 80, No. 18, 1961.
6. Tse, E. and Bar Shalom, Y., “Generalized Certainty Equivalence and Dual Effect in Stochastic Control”, *IEEE Trans. Aut. Contr.,* Vol. AC-20, No. 6, 1975, 817.
7. Wonham, W. M., “On the separation theorem of stochastic control”, *SIAM J. Contr.,* Vol. 6, No. 2, 1968, 312.
8. Shinar, J. and Turetsky, V., “What happens when certainty equivalence is not valid? - Is there an optimal estimator for terminal guidance?”, *Ann. Rev. Contr.,* Vol. 27, No 2, 2003, 119.
9, Witsenhausen, H. S., "Separation of estimation and control for discrete time systems," *Proc. IEEE*, Vol. 59, No. 11, 1971, 1557.
10. Shaviv, I. G. and Oshman, Y., "Estimation-Guided Guidance" *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Keystone, CO, August 2006.
11. Hexner, G. and Shima, T., “Stochastic Optimal Control Guidance Law with Bounded Acceleration“, *IEEE Trans. Aerosp. Elect. Syst.,* Vol. 43, No. 1, 2007, 71.
12. Shinar, J., V. Turetsky, and Y. Oshman, “Integrated Estimation/Guidance Design Approach for Improved Homing Against Randomly Maneuvering Targets“, *J. Guidance, Contr., Dynamics*, Vol. 30, No. 1, 2007, 154.
13. Singer, R. A., “Estimating optimal filter tracking performance for manned maneuvering targets”, *IEEE Transactions of Aerospace and Electrical Systems,* Vol. ES-6, No. 4, 1970, pp. 473-483.
14. Shinar, J., “Solution techniques for realistic pursuit-evasion games“, in *Advances in Control and Dynamic Systems*, (C.T. Leondes, Ed.), Vol. 17, Academic Press, N.Y. 1981, 63.
15. Shinar, J. and Glizer, V. Y., “Solution of a delayed information linear pursuit-evasion game with bounded controls” *Int. Game Theory Review,* Vol. 1, No. 3&4, 2000, 197.
16. Shinar, J. and Shima, T., “Non-orthodox guidance law development approach for the interception of maneuvering anti-surface missiles”, *J. Guidance, Contr.,Dynamics,* Vol.25, No. 4, 2002, 658.
17. Shinar, J. and V. Turetsky, "Three-dimensional validation of an integrated estimation/guidance algorithm against randomly maneuvering targets" *J. Guidance, Contr., Dynamics,* Vol. 32, No. 3, 2009, 1034.